

THERMODYNAMICS

4

CHAPTER FOUR

The Second Law
of Thermodynamics



The Second Law of Thermodynamics

According to the first law of thermodynamics, when a system undergoes a complete cycle then the net heat supplied is equal to the net work done.

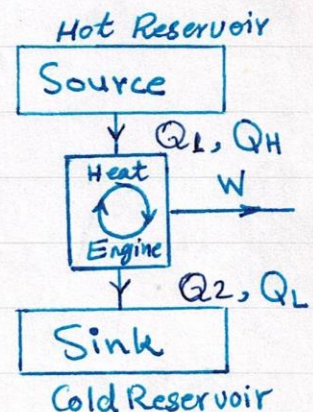
$$\oint dQ = \oint dW$$

The second law of thermodynamics indicates that, although the net heat supplied in a cycle is equal to the net work done, the gross heat supplied must be greater than the net work done, some heat must always be rejected by the system.

• The Heat Engine:

A heat engine is a system operating in a complete cycle and developing net work from a supply of heat. The second law implies that a source of heat supply and a sink for the rejection of heat are both necessary, since some heat must always be rejected by the system.

The heat supplied from the source is Q_1 , the work done is W , and the heat rejected is Q_2 .





By the first law of thermodynamics, in a complete cycle
Net heat supplied = Net work done

$$\oint dQ = \oint dW$$

$$Q_1 - Q_2 = W$$

By the second law, the gross heat supplied must be greater than the net work done.

$$Q_1 > W$$

• Forward Heat Engine:

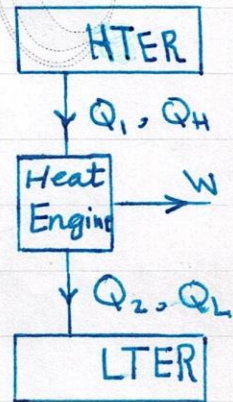
A forward heat engine has a positive work output such as Rankine or Brayton Cycle.

Applying the first law to the cycle gives:

$$Q_1 - Q_2 - W = 0$$

$$Q_1 > W \text{ (heat supplied must be greater than } W)$$

$$Q_2 > 0$$



LTER = Low Temp. Energy Reservoir

• Reverse Heat Engine:

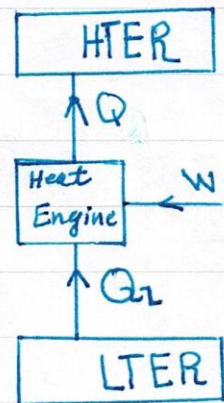
A reverse heat engine has a positive work input such as heat pump and refrigerator.

HTER = High Temp. Energy Reservoir

Applying the first law to the cycle gives:

$$-Q_1 + Q_2 + W = 0$$

$$W > 0$$





• The thermal efficiency:

the thermal efficiency of a heat engine is defined as the ratio of the net work done in the cycle to the gross heat supplied in the cycle.

$$\text{thermal efficiency, } \eta = \frac{W}{Q_1}, \quad W = Q_1 - Q_2$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \Rightarrow \eta < 100\%$$

From the definition of heat, a temperature difference is necessary for heat to flow, therefore that the source of heat must be at a higher temperature than the sink.

the source can be thought of as a hot reservoir and the sink as a cold reservoir.

There are two statements of the Second Law: of thermodynamics

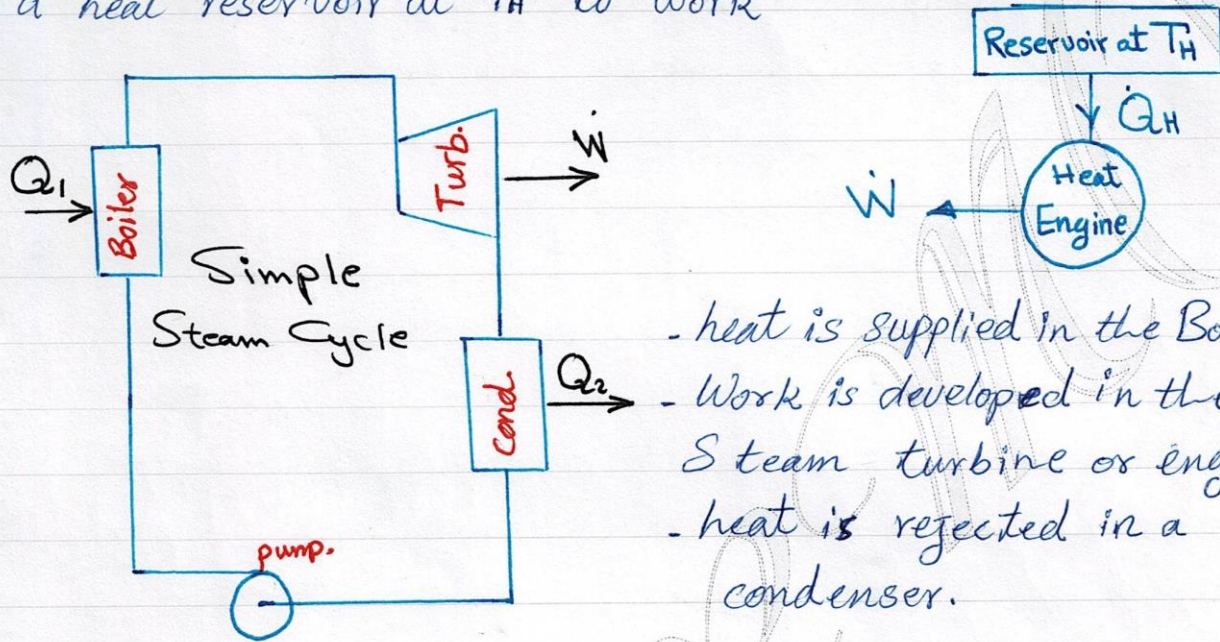
1. The Kelvin-Planck Statement.
2. The Clausius statement.

1. The Kelvin-Planck Statement:

It is impossible for any system to operate in a thermodynamic cycle and deliver a net amount of work to its surroundings while exchanging heat with a single heat reservoir.

The Kelvin-Planck Statement of the 2nd law:
It is not possible to have the following system
with $Q_2 = 0$

We cannot convert all of the heat from a heat reservoir at T_H to work



- heat is supplied in the Boiler
- Work is developed in the Steam turbine or engine
- heat is rejected in a condenser.

Boiler \equiv Hot Reservoir.

Condenser \equiv Cold reservoir.

Steam \equiv System.

• Heat Reservoir:

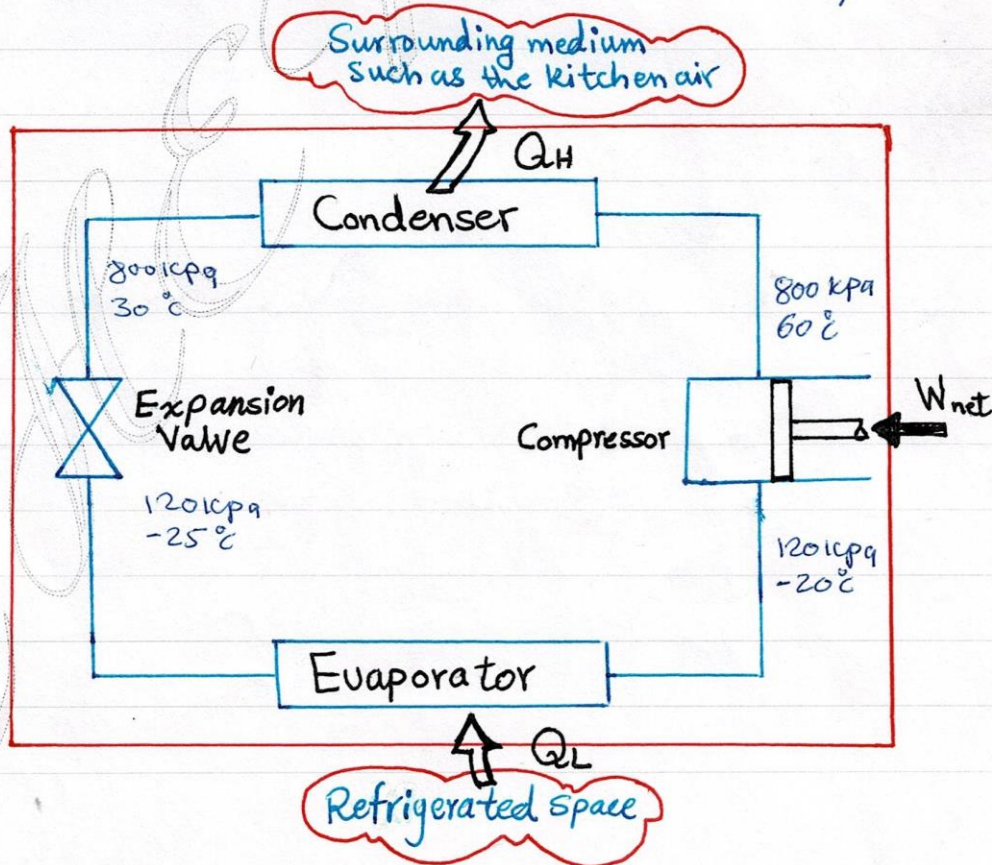
It is a closed system, from which heat is removed, or to which heat is added. The temperature of the reservoir does not change. A heat reservoir may be a large heat source or a large heat sink.

• Refrigerators and Heat Pumps:

We know that the heat flows in the direction of decreasing temperature (from high-temperature media to low-temperature ones), this heat transfer process occurs in nature without requiring any devices.

The reverse process, cannot occur by itself. The transfer of heat from a low-temperature medium to a high-temperature one requires special devices called **refrigerators**.

Refrigerators, like heat engines, are cyclic devices. The working fluid used in the refrigeration cycle is called a **Refrigerant**. This cycle which involves four main components: a compressor, a condenser, an expansion valve, and an evaporator.



• Coefficient of Performance;

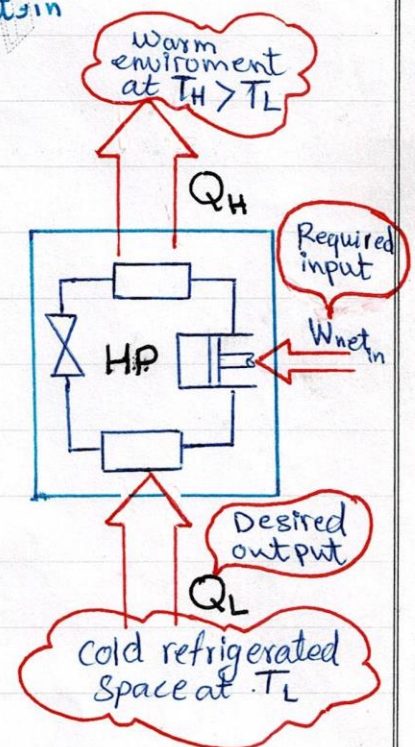
The efficiency of refrigerator is expressed in terms of the coefficient of performance (COP), denoted by COP_R . The objective of a refrigerator is to remove heat (Q_L) from the refrigerated space. To accomplish this objective, it requires a work input of $W_{net,in}$. Then the COP of a refrigerator can be expressed as

$$COP_R = \frac{\text{desired output}}{\text{required input}} = \frac{Q_L}{W_{net,in}}$$

$$W_{net,in} = Q_H - Q_L$$

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1}$$

Notice that the value of COP_R can be greater than unity. That is, the amount of heat removed from the refrigerated space can be greater than the amount of work input.

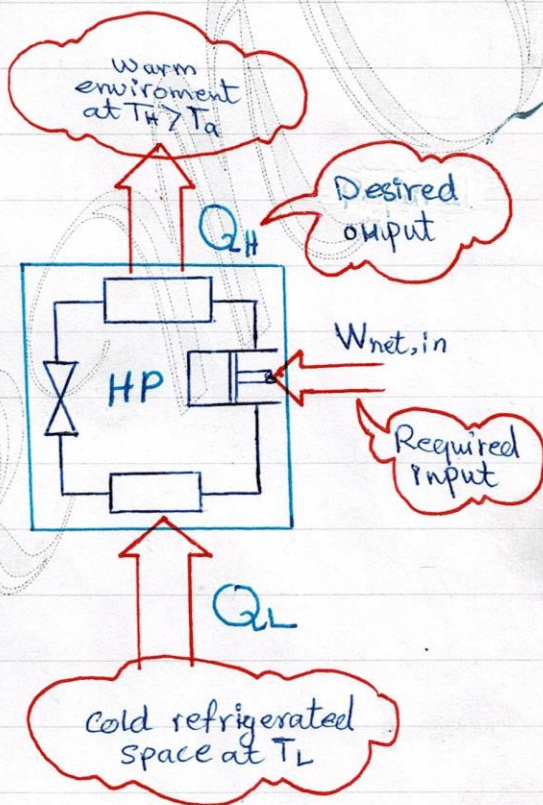


• The Heat Pumps:

It is another device that transfers heat from a low-temperature medium to a high-temperature one.

Refrigerators and heat pumps operate on the same cycle but differ in their objectives.

The measure of performance of heat pump is also expressed in terms of the coefficient of Performance COP_{HP} .



$$COP_{HP} = \frac{\text{desired output}}{\text{required input}} = \frac{Q_H}{W_{net,in}}$$

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - \frac{Q_L}{Q_H}}$$

$$(Q_H = Q_L + W_{net,in})$$

$$COP_{HP} = \frac{Q_L + W_{net,in}}{W_{net,in}}$$

$$= \frac{Q_L}{W_{net,in}} + 1$$

$$COP_{HP} = COP_R + 1$$

2. The Clausius statement:

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower temperature body to a higher temperature body.

Ex: A heat pump is used to meet the heating requirements of a house and maintain it at 20°C . On a day when the outdoor air temperature drops to -2°C , the house is estimated to lose heat at a rate of $80,000 \text{ kJ/h}$. If the heat pump under these conditions has a COP of 2.5, determine (a) the power consumed by the heat pump, (b) the rate at which heat is extracted from cold outdoor air.

Sol:

$$a. \text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{W_{\text{net},\text{in}}}$$

$$W_{\text{net},\text{in}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}} = \frac{80000}{2.5}$$

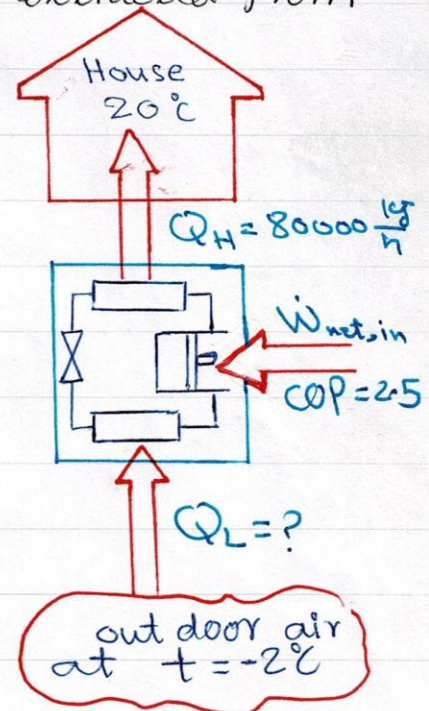
$$= 32000 \text{ kJ/hr}$$

$$= 8.9 \text{ kW}$$

$$b. \dot{Q}_L = \dot{Q}_H - W_{\text{net},\text{in}}$$

$$= 80000 - 32000$$

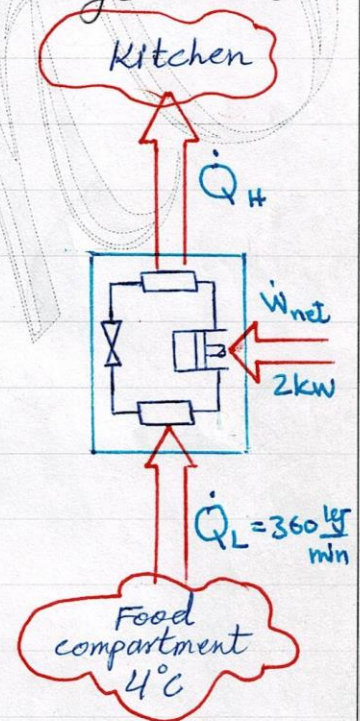
$$= 48000 \text{ kJ/h}$$



• **Ex:** The food compartment of a refrigerator is maintained at 4°C by removing heat from it at rate of 360 kJ/min . If the required power input to the refrigerator is 2 kW , determine a. COP_R b. the rate of heat discharge to the room that houses the refrigerator.

$$\text{Sol: a. } \text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net in}}} = \frac{360 \text{ kJ/min}}{2 \text{ kW}} \left(\frac{1 \text{ kW}}{60 \text{ kJ/min}} \right) = 3$$

$$\begin{aligned} \text{b. } \dot{Q}_H &= \dot{Q}_L + \dot{W}_{\text{net}} \\ &= 360 \frac{\text{kJ}}{\text{min}} + 2 \text{ kW} \left(\frac{60 \text{ kJ/min}}{1 \text{ kW}} \right) \\ &= 480 \text{ kJ/min} \end{aligned}$$



• **Ex:** Heat is transferred to a heat engine from a furnace at a rate of 80 MW . If the rate of waste heat rejection to a nearby river is 50 MW , determine the net power output and the thermal efficiency for this heat engine.

$$\begin{aligned} \text{Sol: } \dot{Q}_H &= 80 \text{ MW}, \quad \dot{Q}_L = 50 \text{ MW} \\ \dot{W}_{\text{net out}} &= \dot{Q}_H - \dot{Q}_L \\ &= 80 - 50 = 30 \text{ MW} \end{aligned}$$

$$\eta_{\text{thermal}} = \frac{\dot{W}_{\text{net out}}}{\dot{Q}_H} = \frac{30}{80} = 0.375 = 37.5\%$$

ENTROPY

The term 'entropy' which literally means transformation, was first introduced by Clausius. It is an important thermodynamics property of a working substance, which increases with the addition of heat, and decreases with its removal.

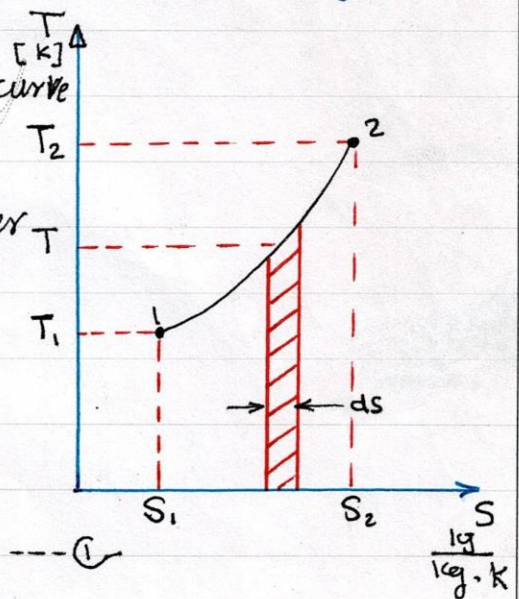
In a reversible process, over a small range of temperature, the increase or decrease of entropy, when multiplied by the absolute temperature, gives the heat absorbed or rejected by the working substance. Mathematically, heat absorbed by the working substance,

Heat transfer reversibly = Area under the curve
 $= T ds$

Total heat transfer $1 \rightarrow 2$ = total area under the curve
 $= \sum_{s=s_1}^{s=s_2} T ds$

in the limit as $ds \rightarrow 0$

Heat transfer reversibly from $1 \rightarrow 2$ = $\int_{s_1}^{s_2} T ds = Q_{rev}$ --- (1)



Differentiating eq. (1)

$$dQ_{rev} = T \cdot ds$$

$$ds = \frac{dQ_{rev}}{T}$$

~~~~ \* (2)



This equation gives the relationship which must exist between  $S$ ,  $T$  and  $Q_{rev}$  in order that the area of the graph shall be heat transferred reversibly.

Now it is possible to calculate the amount of heat transferred reversibly during a non-flow process. Thus, by using equation (2) changes in  $S$  can be determined. It is this function  $S$  which is called **entropy**.

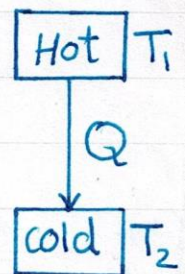
If heat is received, which makes  $Q_{rev}$  positive, then the entropy of the receiving substance has increased.

If heat is rejected, which makes  $Q_{rev}$  negative then the entropy of the rejecting substance is decreased.

Now consider a given quantity of heat energy  $Q$ . Let this quantity of heat be lost from a hot fluid at temperature  $T_1$  and let it be received by a cooler fluid at temperature  $T_2$ .

$$S = \frac{Q}{T} = \frac{Q}{T_1}$$

$$T_1 > T_2 \Rightarrow \frac{Q}{T_2} > \frac{Q}{T_1}$$



$\therefore$  the gain of entropy to the cooler fluid is greater than the loss of entropy from the hot fluid.



\* The entropy of vapours (two-phase system)

8. Liquid entropy:

$$Q = m C_p (T_2 - T_1)$$

$$Q = m C_p \Delta T \quad , \text{ for 1kg mass}$$

$$Q = C_p \Delta T \quad , \text{ for liquid}$$

$$Q = C_{pL} \Delta T \Rightarrow$$

$$dQ = C_{pL} dT \quad \text{--- (1)}$$

$C_{pL}$  = Specific heat capacity of the liquid at constant pressure.  
= 4.187 kJ/kg.K, for H<sub>2</sub>O

Dividing eq.(1) throughout by T, then

$$\frac{dQ}{T} = C_{pL} \frac{dT}{T} = ds \Rightarrow ds = C_{pL} \frac{dT}{T}$$

$$\int_{s_1}^{s_2} ds = C_{pL} \int_{T_1}^{T_2} \frac{dT}{T} \Rightarrow s_2 - s_1 = C_{pL} \ln \frac{T_2}{T_1}$$

When  $T_1 = 0^\circ\text{C} = 273\text{K} \Rightarrow s_1 = 0$

$$s_2 - 0 = C_{pL} \ln \frac{T_2}{273}$$

$$\therefore S = C_{pL} \cdot \ln \frac{T}{273} \quad \text{Liquid Entropy}$$

for H<sub>2</sub>O,  $S = 4.187 \ln \frac{T}{273}$

**Ex:** Determine the value of the specific entropy of water at  $100^\circ\text{C}$ .

**Sol:**  $S = 4.187 \ln\left(\frac{T}{273}\right)$   
 $S_f = 4.187 \ln\frac{373}{273} = 1.31 \text{ kJ/kg}\cdot\text{K}$

from Steam tables at  $100^\circ\text{C}$ ,  $S_f = 1.3068 \text{ kJ/kg}\cdot\text{K}$

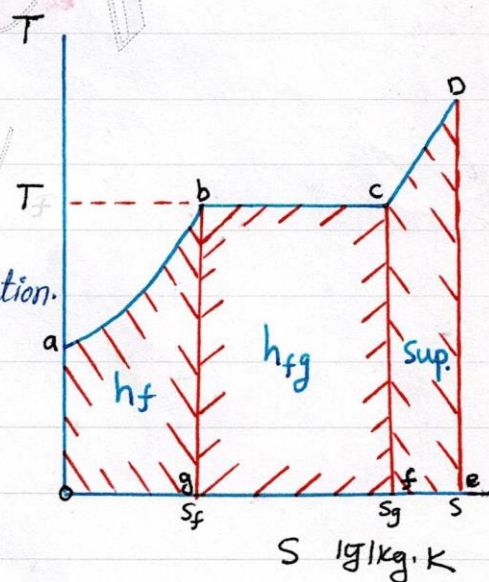
## Entropy of evaporation:

Heat transferred = Change of enthalpy

Area abgo =  $h_f$  = Specific liquid enthalpy

Area bctfg =  $h_{fg}$  = Specific enthalpy of evaporation.

Area cdef =  $sup$  = Specific superheat.



Area bctfg =  $h_{fg} = T_s(S_g - S_f)$

$S_g - S_f = \frac{h_{fg}}{T_s} = \text{Specific entropy of evaporation} = S_{fg}$

for wet steam

$S = S_f + x S_{fg}$  ,  $S_f = 4.187 \ln\frac{T_s}{273}$  for  $\text{H}_2\text{O}$

$S = C_{pL} \cdot \ln\frac{T_s}{273} + x \cdot \frac{h_{fg}}{T_s}$

for wet steam  
at  $x=1 \Rightarrow S = S_g$



**Ex:** Determine the value of the specific entropy of wet steam at a pressure of 2 Mpa and 0.8 dry.

a. by calculation      b. by using values of entropy from Steam tables.

**Sol:** a.  $S = C_p \cdot \ln \frac{T_s}{273} + x \frac{h_{fg}}{T_s}$

from Steam tables at  $p = 2 \text{ Mpa}$ ,  $T_s = 212.42^\circ\text{C} = 485.4 \text{ K}$

$$h_{fg} = 1890.74$$

$$\therefore S = 4.187 \ln \frac{485.4}{273} + 0.8 \times \frac{1890.74}{485.4}$$

$$= 5.525 \text{ kJ/kg}\cdot\text{K}$$

b.  $S = S_f + x S_{fg}$       at 2 Mpa,  $S_f = 2.4473 \text{ kJ/kg}\cdot\text{K}$   
 $= 2.4473 + 0.8 \times 3.8935$        $S_{fg} = 3.8935 \text{ kJ/kg}\cdot\text{K}$   
 $= 5.562 \text{ kJ/kg}\cdot\text{K}$

## Entropy of Superheated Vapour:

$$\text{Area cdef} = C_{pv} \cdot dT$$

$C_{pv}$  = Specific heat capacity of superheated vapour at constant pressure.

$$dQ = C_{pv} \cdot dT \quad \} \div T$$

$$\frac{dQ}{T} = C_{pv} \cdot \frac{dT}{T} \Rightarrow ds = C_{pv} \cdot \frac{dT}{T}$$



$$s_g \int ds = C_{pv} \int_{T_s}^T \frac{dT}{T} \Rightarrow S - S_g = C_{pv} \cdot \ln \frac{T}{T_s}$$

$$S = S_g + C_{pv} \ln \frac{T}{T_s}$$

$$S = C_{pl} \cdot \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_{pv} \ln \frac{T}{T_s}$$

**Ex:** Determine the value of Specific entropy of Steam at 1.5 Mpa with a temperature of 300°C.  $C_{pv} = 2.0934 \text{ kJ/kg} \cdot \text{K}$   
a. by calculation                      b. from Steam tables

**Sol:** a.  $S = C_{pl} \cdot \ln \frac{T_s}{273} + \frac{h_{fg}}{T_s} + C_{pv} \ln \frac{T}{T_s}$

from Steam tables at 1.5 Mpa,  $T_s = 198.32^\circ\text{C} = 471.32 \text{ K}$   
 $h_{fg} = 1947.28 \text{ kJ/kg}$

$$S = 4.187 \cdot \ln \frac{471.32}{273} + \frac{1947.28}{471.32} + 2.0934 \cdot \ln \frac{573}{471.32}$$

$$= 6.8268 \text{ kJ/kg} \cdot \text{K}$$

b. from Superheated Steam tables at 1.5 Mpa & 300°C

$$S = 6.9188 \text{ kJ/kg} \cdot \text{K}$$

|       | 1400 kpa                          | (1500)                                 | 1600 kpa |
|-------|-----------------------------------|----------------------------------------|----------|
| 300°C | 6.9533                            | S                                      | 6.8844   |
|       | $\frac{1500 - 1400}{1600 - 1400}$ | $= \frac{S - 6.9533}{6.8844 - 6.9533}$ |          |

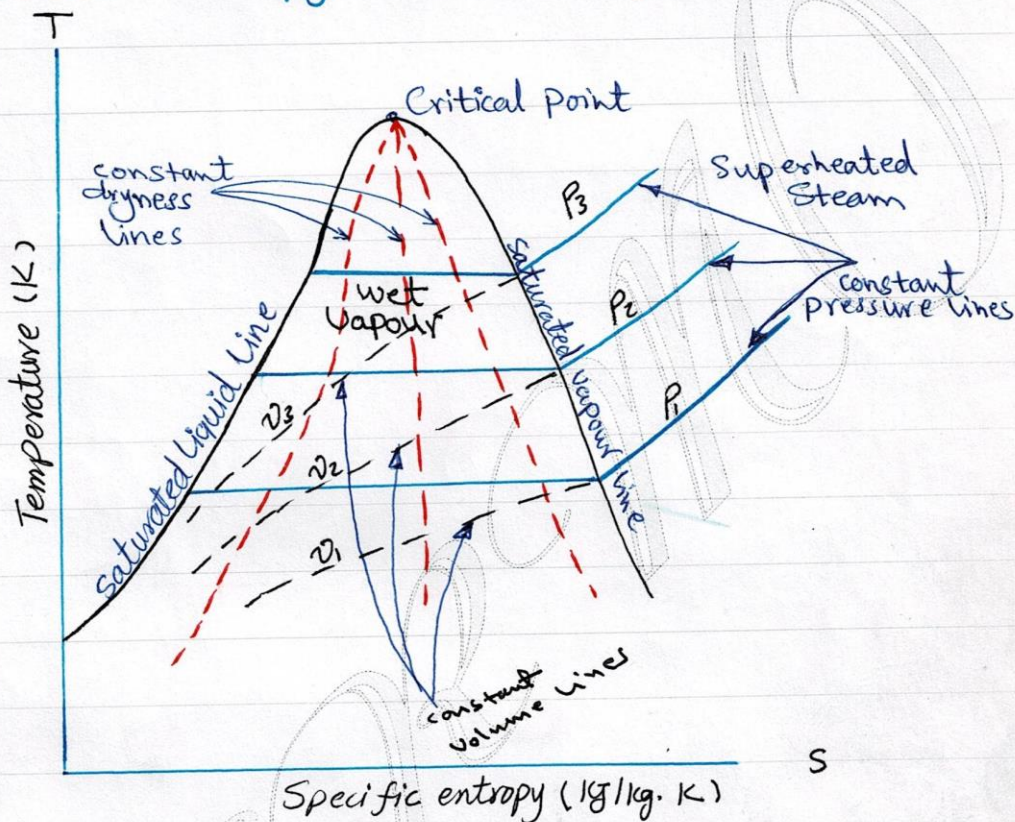


(The temperature - entropy chart for vapour)

$$P_1 < P_2 < P_3$$

$$v_1 > v_2 > v_3$$

$$S_x = S_f + x S_{fg}$$



**Ex:** 1 kg of steam at 7 bar, entropy 6.5 kJ/kg.K is heated reversibly at constant pressure until the temperature is 250°C. Calculate the heat supplied and show on T-s diagram the area which represent the heat flow.

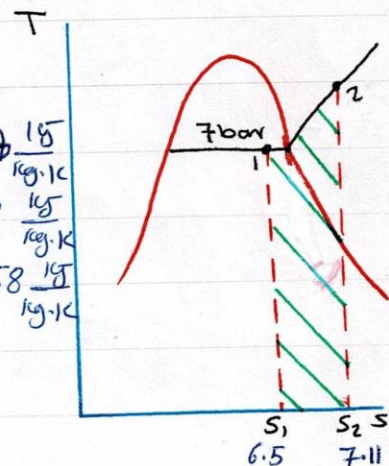
**Sol:**

at 7 bar from steam tables,  $S_g = 6.708 \frac{\text{kJ}}{\text{kg.K}}$   
 $S_1 < S_g \Rightarrow$  wet steam,  $S_f = 1.9922 \frac{\text{kJ}}{\text{kg.K}}$   
 $S_{fg} = 4.7158 \frac{\text{kJ}}{\text{kg.K}}$

$$S_1 = S_f + x S_{fg}$$

$$6.5 = 1.9922 + x \cdot 4.7158$$

$$x = 0.9558$$



$$h_1 = h_f + x \cdot h_{fg} \quad , h_f = 697.2 \text{ kJ/kg} \quad , h_{fg} = 2066.3 \text{ kJ/kg}$$

$$h_1 = 697.2 + 0.9558 \times 2066.3 = 2672.169 \text{ kJ/kg}$$

at point ②  $p = 7 \text{ bar}$  ,  $t = 250^\circ\text{C}$

from Steam tables  $t_s = 164.97^\circ\text{C}$  ,  $t > t_s \Rightarrow$  Superheated  
from Superheated Steam tables ,  $h_2 = 2953.565 \text{ kJ/kg}$

|                                   | 600 kPa | 700 kPa | 800 kPa |
|-----------------------------------|---------|---------|---------|
| $h \rightarrow 250^\circ\text{C}$ | 2957.16 | $h_2$   | 2949.97 |
| $s \rightarrow 250^\circ\text{C}$ | 7.1816  | $s_2$   | 7.0384  |

$$Q = H_2 - H_1 = m(h_2 - h_1)$$

$$= 1(2953.565 - 2672.169) = 281.396 \text{ kJ}$$

$$s_2 = 7.11 \text{ kJ/kg}\cdot\text{K}$$

**Ex:** A rigid cylinder of volume  $0.025 \text{ m}^3$  contains steam at 80 bar and  $350^\circ\text{C}$  the cylinder is cooled until the pressure is 50 bar calculate the state of the steam after cooling and amount of heat rejected by the steam. sketch the process on T-s diagram.

**Sol:**

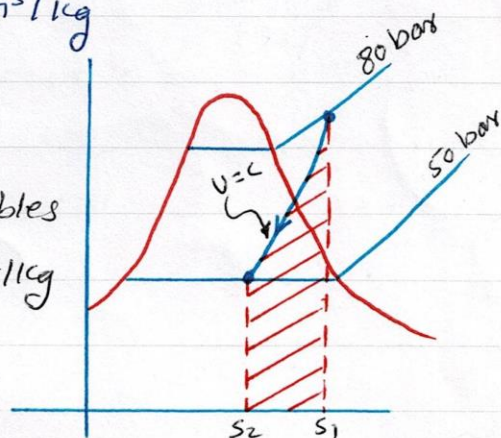
at  $p = 80 \text{ bar}$   $t_s = 295.05^\circ\text{C}$   $t > t_s \Rightarrow$  Superheated  
from Steam table ,  $v_1 = 0.02995 \text{ m}^3/\text{kg}$

$$v_1 = \frac{V_1}{m} \Rightarrow m = \frac{0.025}{0.02995} = 0.8347 \text{ kg}$$

at  $p_1 = 80 \text{ bar}$  &  $350^\circ\text{C}$  from Steam tables

$$h_1 = 2987.3 \text{ kJ/kg} \quad , \quad u_1 = 2747.67 \text{ kJ/kg}$$

$$v_1 = v_2 \quad (\text{rigid cylinder})$$





$\therefore v_2 = 0.02995 \text{ m}^3/\text{kg}$  , at  $P_2 = 50 \text{ bar}$  ,  $v_g = 0.03944 \text{ m}^3/\text{kg}$

$\therefore v_g > v_2 \Rightarrow \text{wet steam}$

$v_2 = x v_g \Rightarrow 0.02995 = x_2 * 0.03944 \Rightarrow x_2 = 0.759$

at 50 bar ,  $u_f = 1147.78 \text{ kJ/kg}$  ,  $u_{fg} = 1449.34 \text{ kJ/kg}$

$u_2 = u_f + x u_{fg} = 1147.78 + 0.759 * 1449.34$   
 $= 2247.82 \text{ kJ/kg}$

$Q = W + \Delta U$  ,  $V = C \Rightarrow W = 0$

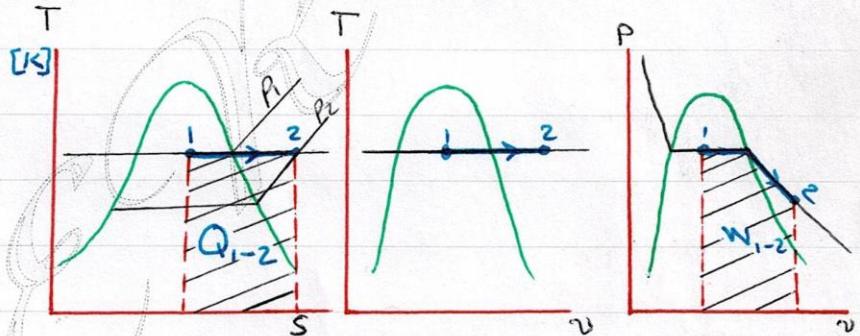
$= m(u_2 - u_1) = 0.8347(2247.82 - 2747.67)$

$= -417.22 \text{ kJ}$  (rejected heat)

\* Isothermal process on T-s chart:

$Q = T(S_2 - S_1)$

$T_1 = T_2$

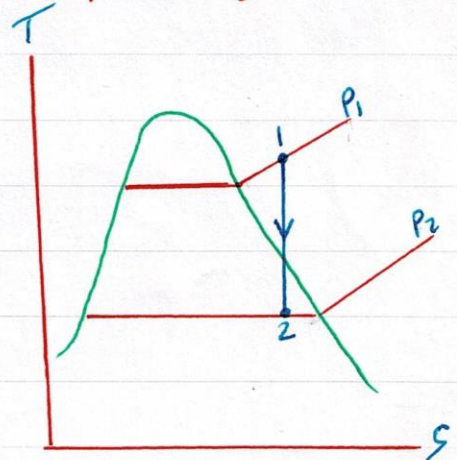


\* Isentropic process (reversible adiabatic process)

$Q = 0$  ,  $S_1 = S_2$

$dQ = T ds$  ,  $s = \text{constant}$

$\therefore Q = 0$   $\therefore ds = 0$



## \* The Entropy of Ideal gas (single-phase system)

$dQ = dW + dU$  for non-flow process  
for unit mass of gas and let its state change from  $P_1, T_1, v_1$  to new state  $P_2, T_2, v_2$

$$dQ = dW + dU \quad \text{for unit mass}$$

$$dQ = dU + dW$$

$$dQ = C_v dT + P dv \quad \div T \quad \text{--- ①}$$

$$\frac{dQ}{T} = C_v \frac{dT}{T} + \frac{P}{T} dv \quad \text{--- ②}$$

$$Pv = RT \Rightarrow P/T = R/v \quad \text{--- ③}$$

Sub. eq (3) in (2)

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v}$$

$$\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} C_v \frac{dT}{T} + \int_{v_1}^{v_2} R \frac{dv}{v} \quad \text{--- ④}$$

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- ⑤}$$

eq (5) represent the relation between (S, T, v).

$C_p - C_v = R$ , substituting this into eq. 5

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \frac{v_2}{v_1} \quad \text{--- ⑥}$$

$$S_2 - S_1 = C_v \left( \ln \frac{T_2}{T_1} - \ln \frac{v_2}{v_1} \right) + C_p \ln \frac{v_2}{v_1} \quad \text{--- ⑦}$$

$$S_2 - S_1 = C_v \ln \left( \frac{T_2}{T_1} \cdot \frac{v_1}{v_2} \right) + C_p \ln \frac{v_2}{v_1} \quad \text{--- (8)}$$

$$\therefore \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \Rightarrow \frac{T_2}{T_1} \cdot \frac{v_1}{v_2} = \frac{P_2}{P_1} \quad \text{--- (9)}$$

Sub. eq. (9) in (8)

$$S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1} \quad \text{--- (10)}$$

eq. (10) represent the relation between (S, P, v)

$C_v = C_p - R$ , sub. in eq (5)

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- (5)}$$

$$\therefore S_2 - S_1 = (C_p - R) \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- (11)}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \left( \ln \frac{T_2}{T_1} - \ln \frac{v_2}{v_1} \right) \quad \text{--- (12)}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \left( \frac{T_2}{T_1} \cdot \frac{v_1}{v_2} \right) \quad \text{--- (13)}$$

$$\therefore \frac{T_2}{T_1} \cdot \frac{v_1}{v_2} = \frac{P_2}{P_1}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{--- (14)}$$

eq. (14) represent the relation between (S, T, P)

## • Isometric Process $V=C$

from eq (5) & (10)

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- (5)}$$

$$v_2 = v_1 \Rightarrow v_2/v_1 = 1 \Rightarrow \ln \frac{v_2}{v_1} = \ln 1 = 0$$

$$\therefore S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$$

and  $S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1} \quad \text{--- (10)}$

$$v_2 = v_1 \Rightarrow \ln v_2/v_1 = 0$$

$$\therefore S_2 - S_1 = C_v \ln \frac{P_2}{P_1}$$

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} = C_v \ln \frac{P_2}{P_1} \quad \text{for constant volume}$$

## • Isobaric process $P=C$

from eq (10) & (14)

$$S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{v_2}{v_1} \quad \text{--- (10)}$$

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{--- (14)}$$

But  $P_1 = P_2 \Rightarrow \ln (P_2/P_1) = 0$ , Sub. in eq (10) & (14)

$$S_2 - S_1 = C_p \ln \frac{v_2}{v_1} = C_p \ln \frac{T_2}{T_1} \quad \text{for constant pressure}$$

بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِیْمِ  
{ وَاكْبَرُ صَمٌّ لِّلْفَقِّ كَارِهُونَ }  
صدق الله بعباده

T  
H  
E  
R  
M  
O  
D  
Y  
N  
A  
M  
I  
C  
S



• Isothermal process  $T=C$

from eqs 5 & 14

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad \text{--- (3)}, \quad T_1 = T_2 \Rightarrow \ln \frac{T_2}{T_1} = 0$$

$$S_2 - S_1 = R \ln \frac{v_2}{v_1}$$

and

$$S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad \text{--- (4)}, \quad \ln \frac{T_2}{T_1} = 0$$

$$S_2 - S_1 = -R \ln \frac{P_2}{P_1} = R \ln \frac{P_1}{P_2}$$

$$S_2 - S_1 = R \ln \frac{v_2}{v_1} = R \ln \frac{P_1}{P_2} \quad \text{for constant temperature}$$

• Polytropic Process  $Pv^n = C$

Heat transferred =  $\frac{\gamma - n}{\gamma - 1}$  . Work done

$$dQ = \frac{\gamma - n}{\gamma - 1} P dv \quad \} \div T$$

$$\frac{dQ}{T} = ds = \frac{\gamma - n}{\gamma - 1} \frac{P dv}{T}$$

$$Pv = RT \Rightarrow P/T = R/v$$

$$ds = \frac{\gamma - n}{\gamma - 1} \cdot R \cdot \frac{dv}{v}$$

$$s_1 \int ds = \frac{\gamma - n}{\gamma - 1} R \int_{v_1}^{v_2} \frac{dv}{v}$$



$$S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} R \ln \frac{v_2}{v_1} \quad , R = C_v(\gamma - 1)$$

$$S_2 - S_1 = \frac{\gamma - n}{\gamma - 1} \cdot C_v(\gamma - 1) \ln \frac{v_2}{v_1}$$

$$S_2 - S_1 = C_v(\gamma - n) \ln \frac{v_2}{v_1} \quad , \frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$S_2 - S_1 = C_v(\gamma - n) \ln \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$S_2 - S_1 = C_v \frac{\gamma - n}{n - 1} \cdot \ln \frac{T_1}{T_2}$$

$$S_2 - S_1 = C_v(\gamma - n) \ln \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}$$

$$S_2 - S_1 = C_v \frac{\gamma - n}{n} \ln \frac{P_1}{P_2}$$

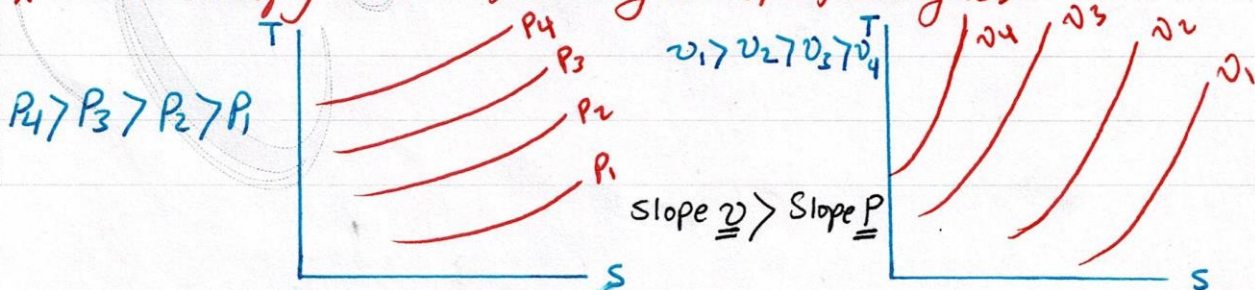
and

$$S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} = C_p \ln \frac{v_2}{v_1} + C_v \ln \frac{P_2}{P_1} = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

• Isentropic process  $S = C$

$$S_2 - S_1 = 0$$

\* The entropy chart for a gas (perfect gas)





**Exo:** Air at  $15^\circ\text{C}$  and  $1.05\text{ bar}$  occupies  $0.02\text{ m}^3$ . The air is heated at constant volume until the pressure is  $4.2\text{ bar}$ , and then cooled at constant pressure back to the original temperature. Calculate the heat flow to or from the air and the net entropy change. Sketch the processes on  $T$ - $S$  diagram.

**Sol:**

$$R = 0.287 \text{ kJ/kg}\cdot\text{K}$$

$$C_v = 0.718 \text{ kJ/kg}\cdot\text{K}, \quad C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$$

$$P_1 V_1 = m R T_1, \quad T_1 = 15 + 273 = 288 \text{ K}$$

$$1.05 \times 10^5 \times 0.02 = m \times 0.287 \times 288$$

$$m = 0.0254 \text{ kg}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow T_2 = \frac{P_2 \cdot T_1}{P_1} = \frac{4.2 \times 288}{1.05} = 1152 \text{ K}$$

$$Q_{1-2} = m C_v (T_2 - T_1) = 0.0254 \times 0.718 \times (1152 - 288) = 15.75 \text{ kJ}$$

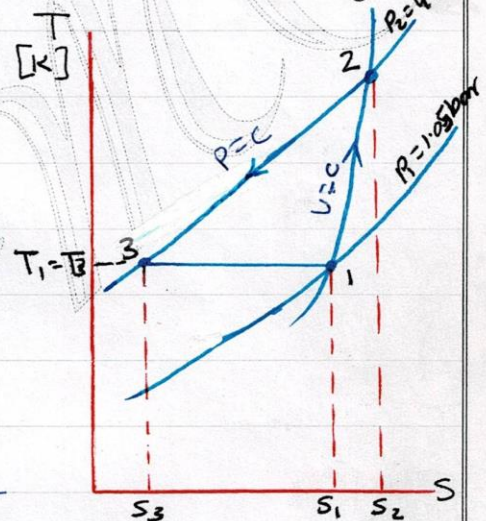
$$Q_{2-3} = m C_p (T_3 - T_2) = 0.0254 \times 1.005 \times (288 - 1152) = -22.05 \text{ kJ}$$

$$Q_{1 \rightarrow 3} = Q_{1-2} + Q_{2-3} = 15.75 + (-22.05) = -6.3 \text{ kJ}$$

for  $U=C$ ,  $S_2 - S_1 = C_v \ln \frac{T_2}{T_1}$   $\text{kJ/kg}\cdot\text{K}$

$$S_2 - S_1 = m C_v \ln \frac{T_2}{T_1} \text{ kJ/K}$$

$$S_2 - S_1 = 0.0254 \times 0.718 \times \ln \frac{1152}{288} = 0.0253 \text{ kJ/K}$$



for  $p=c$  from 2  $\rightarrow$  3

$$S_3 - S_2 = m C_p \ln \frac{T_3}{T_2} = 0.0254 \times 1.005 \times \ln \frac{288}{1152}$$

$$= -0.0354 \text{ kJ/K}$$

$$\Delta S_{\text{net}} = \Delta S_{1-2} + \Delta S_{2-3}$$

$$= 0.0253 - 0.0354$$

$$= -0.0101 \text{ kJ/K}$$

**Ex:**  $0.03 \text{ m}^3$  of nitrogen contained in a cylinder behind a piston is initially at 1.05 bar and  $15^\circ\text{C}$ . The gas is compressed isothermally and reversibly until the pressure is 4.2 bar. Calculate the change of entropy, the heat flow, the work done. Sketch the process on  $p-v$  &  $T-s$  diagram.

**Sol:**  $R = R_0/M = 8.314/28 = 0.297 \text{ kJ/kg}\cdot\text{K}$

$$P_1 V_1 = m R T_1 \quad , \quad T_1 = 15 + 273 = 288 \text{ K}$$

$$1.05 \times 10^2 \times 0.03 = m \times 0.297 \times 288$$

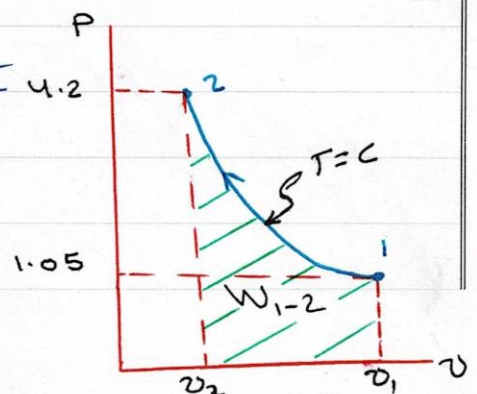
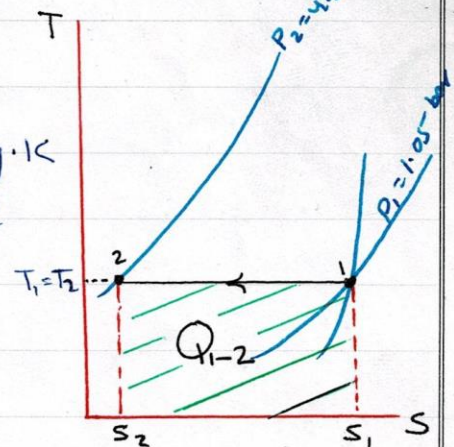
$$m = 0.0368 \text{ kg}$$

$$S_2 - S_1 = m C_p \ln \frac{T_2}{T_1} + m R \ln \frac{P_1}{P_2}$$

$$\therefore S_2 - S_1 = m R \ln \frac{P_1}{P_2} = 0.0368 \times 0.297 \times \ln \frac{1.05}{4.2} \text{ kJ}$$

$$= -0.01516 \text{ kJ/K}$$

$Q_{1-2}$  = area under the line representing the process 1  $\rightarrow$  2



$$Q = T_1(S_2 - S_1) = 288 \times -0.01516 = -4.37 \text{ kJ}$$

$$Q = W + \Delta U, \quad T=C \Rightarrow \Delta U=0$$

$$\therefore Q = W$$

$$\Rightarrow W = -4.37 \text{ kJ}$$

## Entropy & Irreversibility

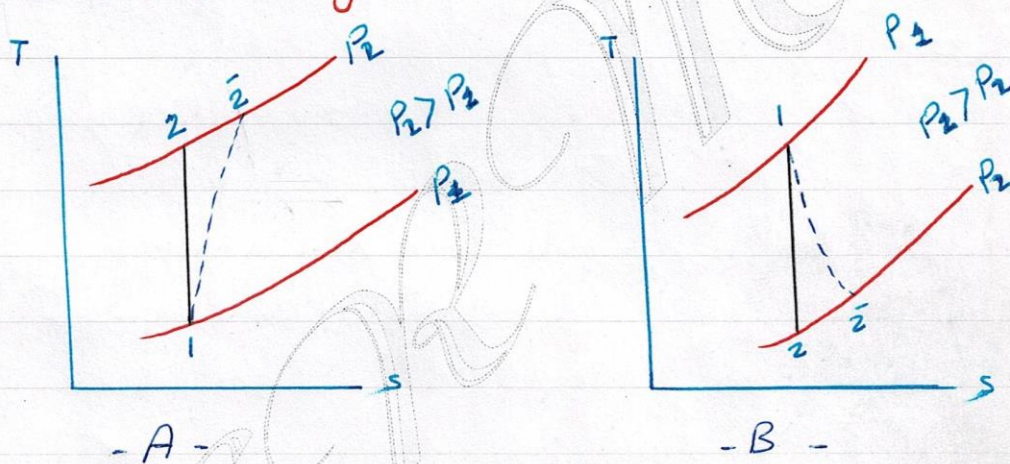


fig. A represent the compression of fluid.

fig. B represent the expansion of fluid.

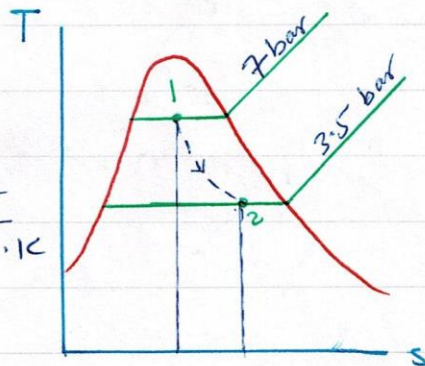
**Ex:** Steam at 7 bar, dryness fraction 0.96, is throttled down to 3.5 bar, calculate the change of entropy per kg of steam.

**Sol:**

$$\text{at 7 bar, } S_1 = S_f + x S_{fg}$$

$$\therefore S_1 = 1.99 + 0.96 \times 4.717 = 6.522 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

throttling  $\Rightarrow h_1 = h_2$



$$h_1 = h_f + x h_{fg} = 697 + 0.96 \cdot 2067$$

$$= 2682 \text{ kJ/kg}$$

$\therefore h_2 = 2682 \text{ kJ/kg}$ , from Steam tables at  $p = 3.5 \text{ bar}$

$$h_g = 2732 \text{ kJ/kg}, h_f = 584 \text{ kJ/kg}$$

$h_f < h_2 < h_g \Rightarrow$  wet Steam

$$h_2 = h_f + x h_{fg} \Rightarrow 2682 = 584 + x_2 \cdot 2148 \Rightarrow x_2 = 0.977$$

$$s_2 = s_f + x_2 s_{fg}, \text{ at } p = 3.5 \text{ bar}$$

$$s_2 = 1.727 + 0.977 \cdot 5.214$$

$$= 6.817 \text{ kJ/kg} \cdot \text{K}$$

**Ex:** In an air turbine the air expands from 6.8 bar and  $430^\circ\text{C}$  to 1.013 bar and  $150^\circ\text{C}$ . The heat loss from the turbine can be assumed to be negligible. Show that the process is irreversible, and Calculate the change of entropy.  $\gamma = 1.4$

**Sol:**

since the process is with negligible heat loss,

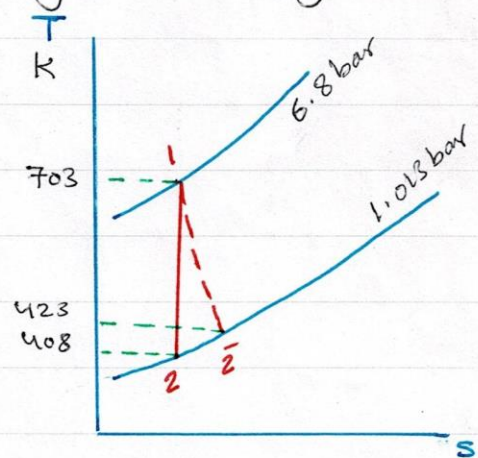
$\therefore$  the process is adiabatic,

If it is adiabatic & reversible,

$\therefore$  it is isentropic

$$T_1 = 430 + 273 = 703 \text{ K}$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow \frac{703}{T_2} = \left(\frac{6.8}{1.013}\right)^{\frac{0.4}{1.4}} \Rightarrow T_2 = 408 \text{ K} = 135^\circ\text{C}$$





the process is irreversible because  $T_2 < \bar{T}_2$   
(If  $T_2 = \bar{T}_2 \Rightarrow$  reversible process)

$$S_1 - S_2 = S_2 - S_2, \quad S_1 = S_2 \quad (\text{reversible adiabatic})$$

the process  $2 \rightarrow \bar{2}$ ,  $P = C$

$$\therefore S_2 - S_{\bar{2}} = C_p \ln \frac{T_2}{\bar{T}_2} = 1.005 \ln \frac{423}{408} = 0.0355 \text{ kg} \cdot \text{K}$$

If  $\Delta S_{\text{total}} \begin{cases} > 0 & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0 & \text{impossible process} \end{cases}$  } for adiabatic process

H.W:

1. Calculate the change of entropy of 1 kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°C to 1.05 bar. The index of expansion is 1.3.

2. 0.05 kg of carbon dioxide ( $M=44$ ) is compressed from 1 bar & 15°C until the pressure is 8.3 bar and the volume is then 0.004 m<sup>3</sup>. Calculate the change of entropy.

Take:  $C_p = 0.88 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$



## (Sheet No. 6)

Q1:

A rigid cylinder containing  $0.006 \text{ m}^3$  of nitrogen at 1.04 bar,  $15^\circ\text{C}$ , is heated reversibly until the temperature is  $90^\circ\text{C}$ . Calculate the change of entropy and the heat supplied. Sketch the process on T-s diagram. Take  $\gamma = 1.4$ , assume nitrogen is a perfect gas.

Ans. [  $0.00125 \frac{\text{kJ}}{\text{K}}$ ,  $0.407 \text{ kJ}$  ]

Q2:

$1 \text{ m}^3$  of air is heated reversibly at constant pressure from  $15^\circ\text{C}$  to  $300^\circ\text{C}$ , and then cooled reversibly at constant volume back to the initial temperature, the initial pressure is 1.03 bar. Calculate the net heat flow and the overall change of entropy, sketch the process on T-s diagram.

Ans. [  $101.5 \text{ kJ}$ ,  $0.246 \text{ kJ/K}$  ]

Q3:

1 kg of air is allowed to expand reversibly in a cylinder behind a piston in such a way that the temperature remains constant at  $260^\circ\text{C}$  while the volume is doubled. The piston is then moved in, and heat is rejected by the air reversibly at constant pressure until the volume is the same as it was initially. Calculate the net heat flow and the overall change of entropy. Sketch the process on T-s diagram.

Ans. [  $-161.9 \frac{\text{kJ}}{\text{kg}}$ ,  $-0.497 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$  ]

Q4: 1 kg of air at 1.02 bar,  $20^\circ\text{C}$  undergoes a process in which the pressure is raised to 6.12 bar, and the volume becomes  $0.25 \text{ m}^3$ . Calculate the change of entropy.

Ans. [  $0.087 \text{ kJ/kg}\cdot\text{K}$  ]



**Q5:** Steam expands reversibly in a cylinder behind a piston from 6 bar dry saturated, to a pressure of 65 kPa. Assuming that the cylinder is perfectly thermally insulated. Calculate the work done during the expansion per kg of steam. **Ans. [323.8 kJ/kg]**

**Q6:** In a centrifugal compressor the air is compressed through a pressure ratio of 4 to 1, and the temperature of the air increases by a factor of 1.65. Show that the process is irreversible and calculate the change of entropy per kg of air. Assume the process is adiabatic. **Ans. [0.105 kJ/kg.K]**

**Q7:** A turbine is supplied with steam at 40 bar, 400°C, which expands through the turbine in steady flow to an exit pressure of 0.2 bar and a dryness fraction of 0.93. The inlet velocity is negligible but the steam leaves at high velocity through a duct of 0.14 m<sup>2</sup> cross-sectional area. If the mass flow rate is 3 kg/s, and the mechanical efficiency is 90%. Calculate the horse power output of the turbine. Show that the process is irreversible and calculate the change of entropy, negligible the heat losses from the turbine.

**Ans. [2018 kW, 0.643 kJ/kg.K]**

## The Heat Engine Cycles

A system is said to have undergone a cycle if it returns to its initial state at the end of the process, and we have two types of cycles  
 1. Thermodynamic cycles      2. Mechanical cycles.

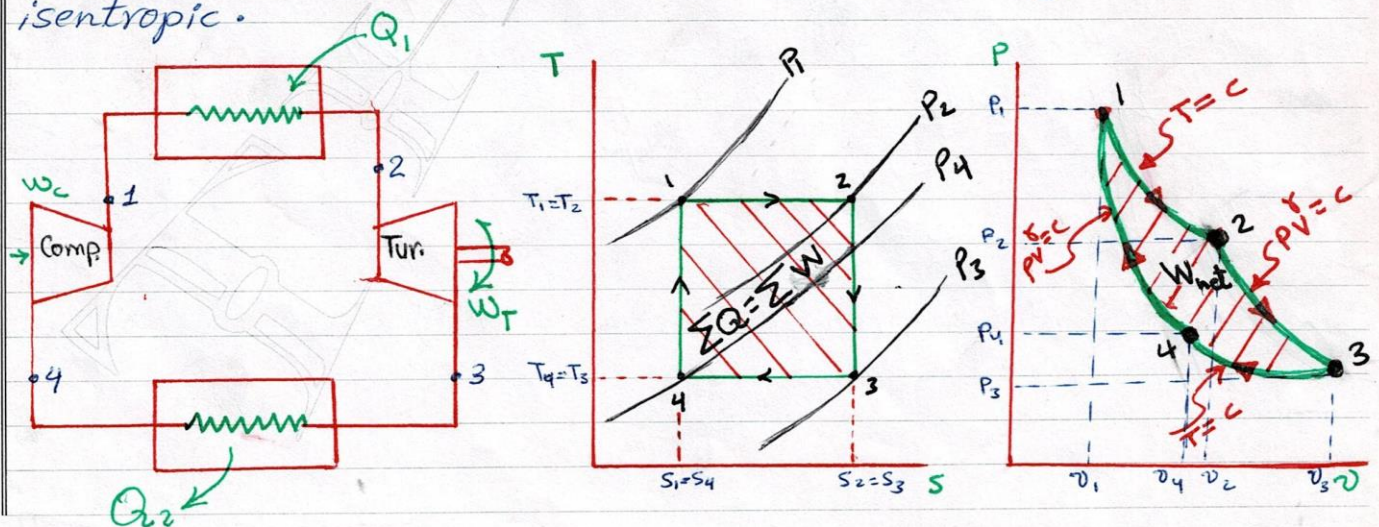
It can be shown that there is an ideal theoretical cycle which is the most efficient conceivable; this cycle is called the **Carnot** cycle. The highest thermal efficiency possible for a heat engine in practice is only about half that of the ideal theoretical Carnot cycle, between the same temperature limits.

### The Carnot cycle:

It can be shown from the Second Law of Thermodynamics that no heat engine can be more efficient than a reversible heat engine working between the same temperature.

Carnot, a French engineer, showed that the most efficient possible cycle is one in which all the heat supplied is supplied at one fixed temperature, and all the heat rejected is rejected at a lower fixed temperature.

The cycle therefore consists of two isothermal processes joined by two adiabatic processes. Since all the processes are reversible, then the adiabatic processes in the cycle are also isentropic.



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\* Carnot Cycle

- Process 1 → 2 : Isothermal process  $T = C$   
from non-flow energy equation

$$Q = W + \Delta U \quad , \quad \text{for gas } \Delta U = 0 \quad \text{when } T = C$$

$$\therefore Q = W = P_1 V_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{V_2}{V_1} \\ = T(S_2 - S_1)$$

- Process 2 → 3 : reversible adiabatic expansion  $PV^\gamma = C$   
(isentropic  $\Delta S = 0$ ,  $S = C$ )

$$Q = W + \Delta U \quad , \quad Q = 0$$

$$\therefore W = -\Delta U = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1} = \frac{mR(T_2 - T_3)}{\gamma - 1}$$

- Process 3 → 4 : Isothermal process, heat rejection  $T = C$

$$Q = W + \Delta U \\ \text{for gas } \Delta U = 0 \quad \text{when } T = C$$

$$Q = W = P_3 V_3 \ln \frac{V_4}{V_3} = mRT_3 \ln \frac{V_4}{V_3} = T(S_3 - S_4)$$

- Process 4 → 1 : reversible adiabatic compression  $PV^\gamma = C$   
(isentropic  $\Delta S = 0$ ,  $S = C$ )

$$Q = W + \Delta U \quad , \quad Q = 0$$

$$W = -\Delta U = \frac{P_4 V_4 - P_1 V_1}{\gamma - 1} = \frac{mR(T_4 - T_1)}{\gamma - 1}$$

Work done by Carnot Cycle,

$$W.D = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

$$= mRT_1 \ln \frac{V_2}{V_1} + \frac{mR(T_2 - T_3)}{\gamma - 1} + mRT_3 \ln \frac{V_4}{V_3} + \frac{mR(T_4 - T_1)}{\gamma - 1}$$

Since  $T_1 = T_2$  and  $T_3 = T_4$

$$\therefore (T_2 - T_3) = (T_1 - T_4)$$

$$W.D = mRT_1 \ln \frac{V_2}{V_1} + \frac{mR(T_2 - T_3)}{\gamma - 1} + mRT_3 \ln \frac{V_4}{V_3} - \frac{mR(T_1 - T_4)}{\gamma - 1}$$

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$$\therefore W.D = mRT_1 \ln \frac{V_2}{V_1} + mRT_3 \ln \frac{V_4}{V_3}$$

Process 4→1  $PV^\delta = C$  , process 2→3  $PV^\delta = C$

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\delta-1} \quad , \quad \frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\delta-1}$$

since  $\frac{T_1}{T_4} = \frac{T_2}{T_3} \Rightarrow \frac{V_4}{V_1} = \frac{V_3}{V_2} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$

$$\therefore W.D = mRT_1 \ln \frac{V_2}{V_1} - mRT_3 \ln \frac{V_2}{V_1}$$

$$W.D = mR(T_1 - T_3) \ln \frac{V_2}{V_1}$$

$$\text{Work ratio} = \frac{\text{Work done}}{\text{Gross work}}$$

thermal efficiency,  $\eta_{th}$

$$\eta_{th} = \frac{\int \delta dQ}{Q_{add}} = \frac{Q_1 - Q_2}{Q_1} = \frac{W.D}{Q_1}$$

$$\eta_{th} = \frac{mR(T_1 - T_3) \ln \frac{V_2}{V_1}}{mRT_1 \ln \frac{V_2}{V_1}} = \frac{T_1 - T_3}{T_1} \Rightarrow$$

$$\eta_{th})_{\text{Carnot}} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_{\min}}{T_{\max}}$$

**Carnot Cycle on T-s diagram:**

1. Heat added =  $mT_1 (s_2 - s_1)$
2. Heat rejected =  $mT_3 (s_2 - s_1)$
3. Work done by the cycle =  $\int \delta dw = \int \delta dQ = Q_1 - Q_2$

$$W.D = mT_1 (s_2 - s_1) - mT_3 (s_2 - s_1) \\ = m(T_1 - T_3)(s_2 - s_1) \quad \text{①}$$

the change of entropy ( $s_2 - s_1$ ) occurs at constant temperature.

$$Q = W + \Delta U \\ dQ = \delta dw + dU = pdv + C_v dT \quad \} \div T$$

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$$\frac{dQ}{T} = p \frac{dV}{T} + C_v \frac{dT}{T} = ds \quad , \text{ since } T=C \Rightarrow dT=0$$

$$ds = \frac{P}{T} dV \quad , \quad pV = mRT \Rightarrow \frac{P}{T} = \frac{mR}{V}$$

$$dS = mR \ln \frac{V_2}{V_1}$$

$$ds = R \ln \frac{V_2}{V_1} \quad \text{--- 2}$$

Sub. eq 2 in 1

$$\therefore W.D = mR(T_1 - T_3) \ln \frac{V_2}{V_1}$$

$$\eta_{th} = \frac{\sum dQ}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{mT_3(S_2 - S_1)}{mT_1(S_2 - S_1)}$$

$$\eta_{th} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_{min}}{T_{max}}$$

$r$  = volume ratio of adiabatic process

$$r = \frac{V_3}{V_2} = \frac{V_4}{V_1} \quad , \quad \frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad , \quad T_1 = T_2$$

$$\eta_{th} = 1 - \frac{T_3}{T_1} = 1 - \frac{T_3}{T_2} \Rightarrow \eta_{th} = 1 - \frac{1}{r^{\gamma-1}}$$

• Types of volume ratio:

1. Volume ratio of adiabatic process:

$$r = \frac{V_3}{V_2} = \frac{V_4}{V_1} > 1$$

2. Volume ratio of isothermal process:

$$r = \frac{V_3}{V_4} = \frac{V_2}{V_1} > 1$$

3. Overall volume ratio:

$$r = \frac{V_3}{V_1} = \frac{\text{max. Volume in Cycle}}{\text{min. Volume in Cycle}}$$

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**Ex:** The overall volume ratio of a Carnot cycle is 15. The temperature limits of the cycle are  $260^\circ\text{C}$  and  $21^\circ\text{C}$ . Determine:  
a. the volume ratio of isothermal and adiabatic process,  
b. the thermal efficiency. (for air)

**Sol:**  $\frac{V_3}{V_1} = 15$

$$r = \frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}}, \quad T_1 = T_2$$

$$\therefore r = \left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}} = \left(\frac{260+273}{21+273}\right)^{\frac{1}{1.4-1}}$$

$$r = 4.425$$

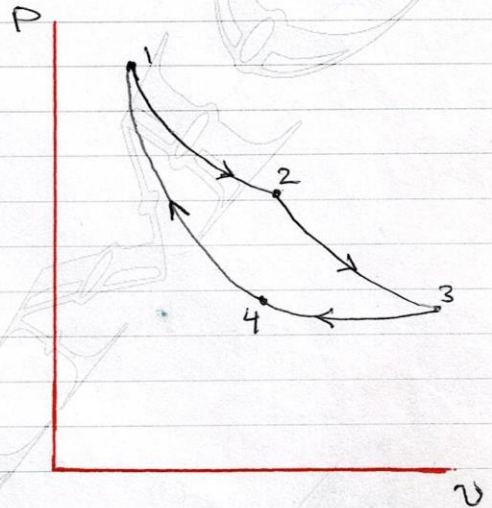
$$\frac{V_2}{V_1} = \frac{V_2}{V_1} \cdot \frac{V_3}{V_3} = \frac{V_2}{V_3} \cdot \frac{V_3}{V_1}$$

$$= \frac{1}{4.425} \cdot 15 = 3.35$$

$$\eta_{th} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{(21+273)}{(260+273)}$$

$$= 0.448$$

$$\therefore \eta_{th} = 44.8\%$$



**Ex:** 1 kg of air is at initial pressure and temperature of 1.73 MPa,  $175^\circ\text{C}$  respectively. It is expanded isothermally to three times of the initial volume, and then further expanded adiabatically to six times of the initial volume. Isothermal compression followed by adiabatic compression returns to its original state.

for this cycle:

- a. name the cycle      b. determine (P, T & V) at each corner of cycle  
c. find  $\eta_{th}$       d. calculate the work done

**Sol:** a. the cycle is Carnot cycle

b. point (1)  $P_1 = 1.73 \text{ MPa}$ ,  $T_1 = 175^\circ\text{C}$

$$P_1 V_1 = mRT_1 \Rightarrow V_1 = mRT_1 / P_1 \Rightarrow V_1 = 0.075 \text{ m}^3$$

Point (2)

$$V_2 = 3V_1 \Rightarrow V_2 = 0.225 \text{ m}^3$$

$$T_1 = T_2 = 175^\circ\text{C}, \quad (T=C)$$

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Point (3)

$$V_3 = 6V_1 \Rightarrow V_3 = 0.45 \text{ m}^3$$

$$P_2 V_2^\gamma = P_3 V_3^\gamma \Rightarrow P_3 = \left(\frac{V_2}{V_3}\right)^\gamma P_2 \Rightarrow P_3 = 0.219 \text{ Mpa}$$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow T_3 = \left(\frac{P_3 V_3}{P_2 V_2}\right) T_2 \Rightarrow T_3 = 67.6^\circ \text{C}$$

Point (4)

$$T_3 = T_4 = 67.6^\circ \text{C} \quad (T=c)$$

$$\left(\frac{T_1}{T_4}\right)^{\frac{1}{\gamma-1}} = \frac{V_4}{V_1} \Rightarrow V_4 = 0.148 \text{ m}^3$$

$$\frac{P_1}{P_4} = \left(\frac{V_4}{V_1}\right)^\gamma \Rightarrow P_4 = 0.656 \text{ Mpa}$$

c.

$$\eta_{th} = 1 - \frac{T_{min}}{T_{max}} \Rightarrow \eta_{th} = 0.242 = 24.2\%$$

d.

$$W.D = m R (T_1 - T_3) \ln \frac{V_2}{V_1} = 34.2 \text{ kJ}$$

**Ex:** A hot reservoir at  $800^\circ \text{C}$  and cold reservoir at  $15^\circ \text{C}$  are available. Calculate the thermal efficiency and the work ratio of a Carnot cycle using air as the working fluid. If the max. & min. pressure in the cycle are 210 bar & 1 bar.

**Sol:**

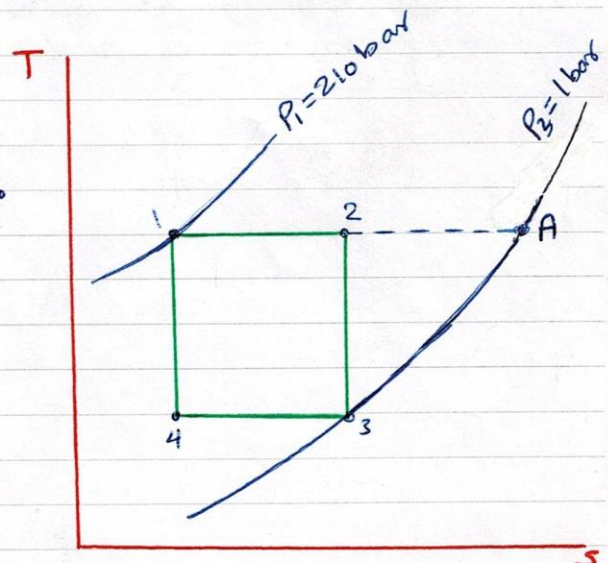
$$\eta_{Carnot} = 1 - \frac{T_3}{T_2} = 1 - \frac{288}{1073} = 0.732 = 73.2\%$$

at  $T=c$ ,

$$s_A - s_1 = R \ln \frac{P_1}{P_3}$$

$$= 0.287 \ln \frac{210}{1}$$

$$= 1.535 \text{ kJ/kg} \cdot \text{K}$$





at  $p=c$

$$S_A - S_3 = C_p \ln \frac{T_2}{T_3} = 1.005 \ln \frac{1073}{288} = 1.34 \text{ kJ/kg} \cdot \text{K}$$

$$\begin{aligned} S_3 - S_1 &= S_2 - S_1 = (S_A - S_1) - (S_A - S_3) \\ &= 1.5353 - 1.321 \\ &= 0.214 \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

$$\oint W = \oint Q = \text{area } 12341 = (T_2 - T_3)(S_2 - S_1), T_2 = T_1$$

$$\begin{aligned} \oint W &= (1073 - 288)(0.214) \\ \therefore W_{\text{net}} (W.D) &= 168 \text{ kJ/kg} \end{aligned}$$

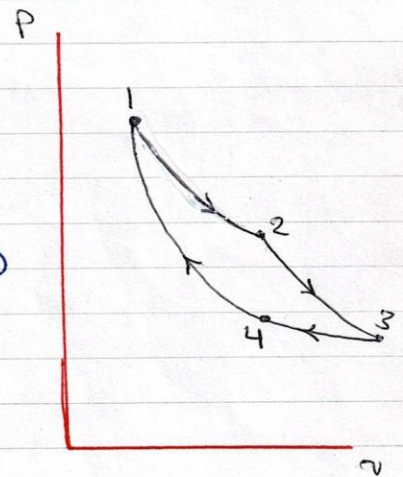
$$W_{\text{gross}} = W_{1-2} + W_{2-3}$$

$$\begin{aligned} W_{1-2} = Q_{1-2} &= T_1(S_2 - S_1), T=c \Rightarrow \Delta U=0 \\ &= 1073 \times 0.214 \\ &= 229.6 \end{aligned}$$

$$\begin{aligned} W_{2-3} = U_2 - U_1 &= C_v(T_2 - T_3), (Q=0, \Delta S=0) \\ &= 0.718(1073 - 288) \\ &= 563.6 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} W_{\text{gross}} &= 229.6 + 563.6 \\ &= 793.2 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Work ratio, (W.R.)} &= \frac{W_{\text{net}}}{W_{\text{gross}}} = \frac{168}{793.2} \\ &= 0.212 \end{aligned}$$

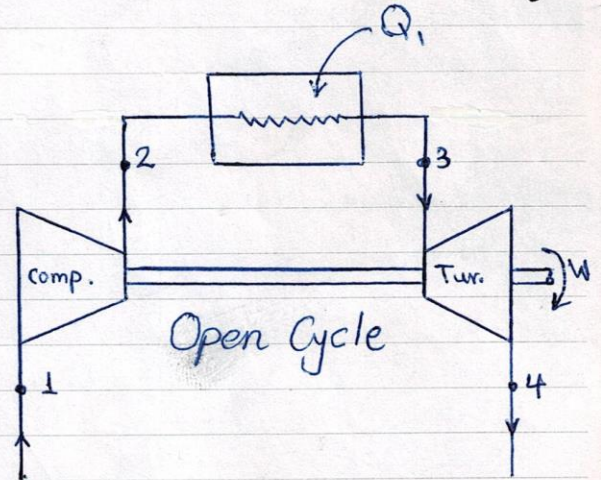
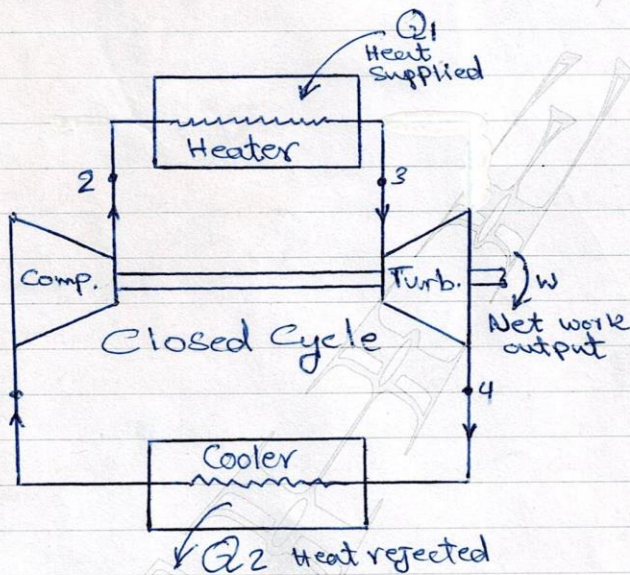
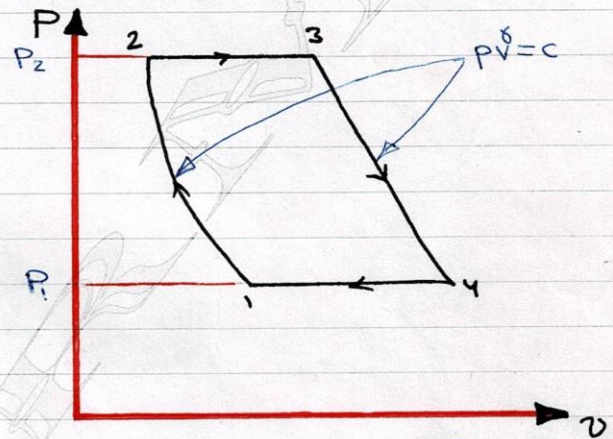
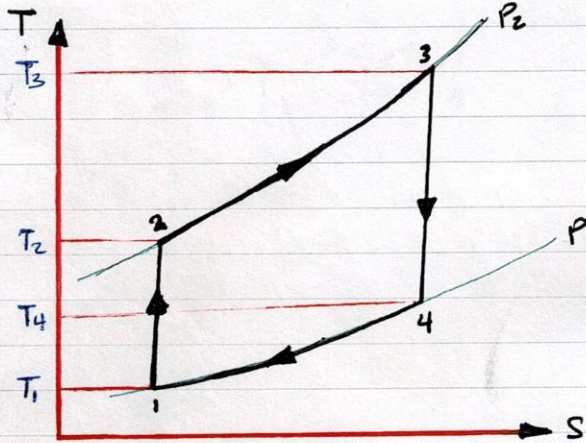


THERMODYNAMICS



## Joule or Brayton cycle (constant pressure cycle)

In this cycle the heat supply and heat rejection processes occur reversibly at constant pressure. The expansion and compression processes are isentropic. This cycle was at one time used as the ideal basis for a hot-air reciprocating engine.



Process 1 → 2 (adiabatic reversible compression)  $PV^\delta = c$

$$gz_1 + h_1 + \frac{1}{2}C_1^2 + q = gz_2 + h_2 + \frac{1}{2}C_2^2 + w$$

$$z_1 = z_2, \quad C_1 = C_2, \quad q = 0$$

$$\therefore \text{Work input to Compressor} = h_1 - h_2 = C_p(T_1 - T_2)$$



Process 2

$$gz_2 + h_2 + \frac{1}{2}C_2^2 + q = gz_3 + h_3 + \frac{1}{2}C_3^2 + w$$

$$z_2 = z_3, \quad C_2 = C_3, \quad w = 0$$

$$\therefore \text{heat supplied to heater} = h_3 - h_2 = C_p(T_3 - T_2)$$

Process 3  $\rightarrow$  4 (adiabatic reversible expansion)  $PV^\gamma = C$

$$gz_3 + h_3 + \frac{1}{2}C_3^2 + q = gz_4 + h_4 + \frac{1}{2}C_4^2 + w$$

$$z_3 = z_4, \quad C_3 = C_4, \quad q = 0$$

$$\therefore \text{Work output from turbine} = h_3 - h_4 = C_p(T_3 - T_4)$$

Process 4  $\rightarrow$  1 (constant pressure heat rejection)  $P = C$

$$gz_4 + h_4 + \frac{1}{2}C_4^2 + q = gz_1 + h_1 + \frac{1}{2}C_1^2 + w$$

$$z_4 = z_1, \quad C_4 = C_1, \quad w = 0$$

$$\therefore \text{heat rejected in cooler} = h_1 - h_4 = C_p(T_1 - T_4)$$

for positive value

$$\text{work comp.} = h_2 - h_1 = C_p(T_2 - T_1)$$

$$\text{heat rejected} = h_4 - h_1 = C_p(T_4 - T_1)$$

$$\begin{aligned} \text{thermal efficiency} &= \frac{\text{Net work}}{\text{heat added}} \\ &= \frac{W_{\text{net}}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \end{aligned}$$

$$\eta_{\text{th}} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \text{--- *}$$

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$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad , \quad \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\because P_1 = P_4 \text{ and } P_2 = P_3 \Rightarrow \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

$$\therefore \frac{T_3}{T_4} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_3 = T_4 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

let  $\frac{P_2}{P_1} = r_p = \text{pressure ratio}$

$$T_3 = T_4 \cdot r_p^{\frac{\gamma-1}{\gamma}} \quad \text{--- ①}$$

$$T_2 = T_1 \cdot r_p^{\frac{\gamma-1}{\gamma}} \quad \text{--- ②}$$

Sub eq 1 & 2 in \* eq.

$$\eta_A = 1 - \frac{T_4 - T_1}{T_4 \cdot r_p^{\frac{\gamma-1}{\gamma}} - T_1 r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{(T_4 - T_1)}{(T_4 - T_1) r_p^{\frac{\gamma-1}{\gamma}}}$$

$$\therefore \eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$$

$$\text{Work Ratio} = \frac{\text{Net work}}{\text{Gross work}} = \frac{W_{Tur.} - W_{comp.}}{W_T}$$

$$W.R = 1 - \frac{C_p(T_2 - T_1)}{C_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$T_2 = T_1 \cdot r_p^{\frac{\gamma-1}{\gamma}} \quad , \quad T_4 = T_3 \cdot \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$$

$$W.R = 1 - \frac{T_1 r_p^{\frac{\gamma-1}{\gamma}} - T_1}{T_3 - T_3 \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}} = 1 - \frac{T_1 (r_p^{\frac{\gamma-1}{\gamma}} - 1)}{T_3 \left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right)}$$

$$W.R = 1 - \frac{T_1}{T_3} \cdot \frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{r_p^{\frac{\gamma-1}{\gamma}} - 1}$$

$$W.R = 1 - \frac{T_1}{T_3} \cdot r_p^{\frac{\gamma-1}{\gamma}}$$

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**Ex:** In a gas turbine unit air is drawn in at 1.02 bar and 15°C and compressed to 6.12 bar. Calculate the thermal efficiency and the work ratio of the ideal constant pressure cycle when the maximum temperature is 800°C.

**Sol:**  $r_p = \frac{P_2}{P_1} = \frac{6.12}{1.02} = 6$

$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \Rightarrow \eta_{th} = 0.4 = 40\%$$

$$T_{min} = T_1 = 15^\circ\text{C}, \quad T_{max} = T_3 = 800^\circ\text{C}$$

$$\text{Work ratio} = 1 - \frac{T_1}{T_3} \cdot r_p^{\frac{\gamma-1}{\gamma}} \Rightarrow \text{Work ratio} = 0.552$$

112

$$T_1 = 15 + 273 = 288\text{K}, \quad T_3 = 800 + 273 = 1073\text{K}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 481\text{K}$$

$$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 643\text{K}$$

$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{1.67} = 0.4 = 40\%$$

$$W_{net} = C_p(T_3 - T_4) - C_p(T_2 - T_1)$$

$$= 1.005(1073 - 643) - 1.005(481 - 288) = 238 \text{ kJ/kg}$$

$$W_{gross} = C_p(T_3 - T_4) = 1.005(1073 - 643) = 432 \text{ kJ/kg}$$

$$\text{Work ratio} = \frac{W_{net}}{W_{gross}} = \frac{238}{432} = 0.552$$

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**Ex:** In an air-standard Brayton cycle the air enters the compressor at 0.1 MPa, 15°C. The pressure leaving the compressor is 1.0 MPa, and the maximum temperature in the cycle is 1100°C. Determine

1. the pressure and temperature at each point in the cycle.
2. the compressor work, turbine work, heat added and  $\eta_{th}$ .

**Sol:** 
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = (15+273) \cdot \left(\frac{1}{0.1}\right)^{\frac{1.4-1}{1.4}} = 556 \text{ K}$$

$$W_{comp} = h_2 - h_1 = C_p(T_2 - T_1) = 1.005(556 - 288) = 269.34 \text{ kJ/kg}$$

$$W_{turbine} = h_3 - h_4 = C_p(T_3 - T_4) \quad , \quad T_3 = 1100^\circ\text{C} = 1373 \text{ K}$$

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = 10^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 710.8 \text{ K}$$

$$W_T = 1.005(1373 - 710.8) = 665.5 \text{ kJ/kg}$$

$$W_{net} = W_T - W_c = 665.5 - 269.34 = 396.16 \text{ kJ/kg}$$

$$q_{add.} = h_3 - h_2 = C_p(T_3 - T_2) = 1.005(1373 - 556) = 821 \text{ kJ/kg}$$

$$\eta_{th} = \frac{W_{net}}{q_{add.}} = \frac{396.16}{821} = 0.482 = 48.2\%$$

or

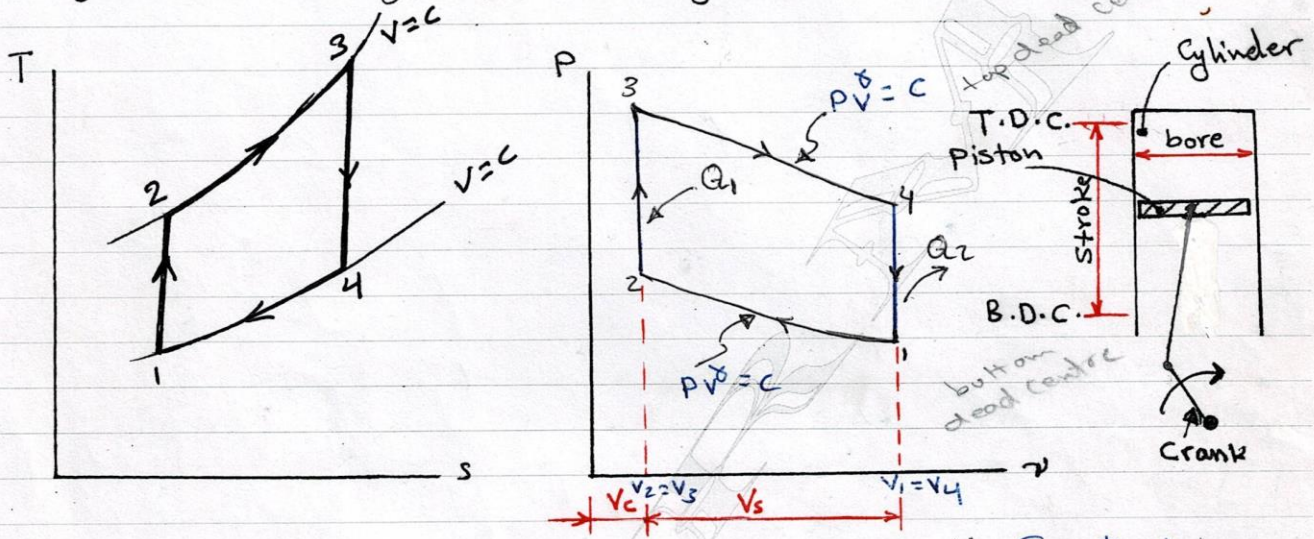
$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{10^{0.286}} = 48.2\%$$

ولذلك أذ ولدتك أمك باكيًا  
والناس حولك يضحكون سرورًا  
فأبعد لنفسك أن تكون أذابكوا  
في يوم موتك ضامكاً سرورًا



## Otto cycle (constant volume cycle)

The Otto cycle is the ideal air standard cycle for the petrol engine, gas engine, and the high-speed oil engine.



$V_s$  = Swept volume  
 $V_c$  = Clearance

**Process 1 → 2** Isentropic compression

$$Q = W + \Delta U, \text{ for } m = 1 \text{ kg}, Q = 0$$

$$W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$$

**Process 2 → 3** Reversible constant volume heat added

$$Q = W + \Delta U, \text{ } W = 0$$

$$Q_1 = \Delta U = u_3 - u_2 = C_v(T_3 - T_2)$$

**Process 3 → 4** Isentropic Expansion

$$Q = W + \Delta U, \text{ } Q = 0$$

$$W = -\Delta U = -(u_4 - u_3) = C_v(T_3 - T_4)$$

**Process 4 → 1** Reversible constant volume heat rejection

$$Q = W + \Delta U, \text{ } W = 0$$

$$Q_2 = \Delta U = u_1 - u_4 = C_v(T_1 - T_4)$$

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for positive vum.

$$W_{comp} = C_v (T_2 - T_1)$$

$$Q_{ref} = C_v (T_4 - T_1)$$

$$r_v = \frac{V_1}{V_2} = \frac{\text{Swept volume} + \text{Clearance volume}}{\text{Clearance volume}}$$

$$\eta_{th} = \frac{\text{Network}}{\text{heat added}} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

$$\eta_{th} = 1 - \frac{C_v (T_4 - T_1)}{C_v (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \frac{T_3}{T_4} = r_v^{\gamma-1}$$

$$T_3 = T_4 \cdot r_v^{\gamma-1} \quad \& \quad T_2 = T_1 \cdot r_v^{\gamma-1}$$

$$\eta_{th} = 1 - \frac{T_4 - T_1}{T_4 r_v^{\gamma-1} - T_1 \cdot r_v^{\gamma-1}} = 1 - \frac{(T_4 - T_1)}{(T_4 - T_1) r_v^{\gamma-1}}$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}}$$

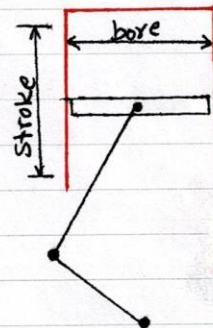
**Ex:** Calculate the ideal air standard thermal efficiency of the Otto cycle for a petrol engine with a cylinder bore of 50 mm and a stroke of 75 mm and clearance volume of 21.3 cm<sup>3</sup>.

**Sol:** Swept Volume = area of bore \* stroke

$$\therefore S.V. = \frac{\pi}{4} (5)^2 * 75 = 147.26 \text{ cm}^3$$

$$r_v = \frac{147.26 + 21.3}{21.3} = 7.914$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}} = 0.56$$



**Ex:** In an air standard Otto cycle the maximum and minimum temperature are  $1400^{\circ}\text{C}$  and  $15^{\circ}\text{C}$ . The heat supplied per kg of air is  $800 \text{ kJ}$ . Calculate the compression ratio and the thermal efficiency. Calculate also the ratio of maximum to minimum pressure in the cycle.

**Sol:**

$$Q_1 = m C_v (T_3 - T_2) \quad \rightarrow T_3 = 1400^{\circ}\text{C} = 1673 \text{ K}$$

$$800 = 1 \times 0.717 (1673 - T_2)$$

$$\therefore T_2 = 557.2 \text{ K} = 284.2^{\circ}\text{C} \quad \rightarrow T_1 = 15^{\circ}\text{C} = 288 \text{ K}$$

$$\frac{T_2}{T_1} = r_v^{\gamma-1} \quad \Rightarrow \quad \frac{557.2}{288} = r_v^{1.4-1}$$

$$r_v = 5.2$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}} = 1 - \frac{1}{5.2^{1.4-1}} = 0.483$$

$$\frac{P_3}{P_1} = ? \quad (\text{H.W.})$$

$$\frac{P_3}{P_1} = 30.24$$

$$\frac{P_3}{P_2} = \frac{P_3}{P_2} \cdot \frac{P_2}{P_1}$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1} \quad \Rightarrow \quad \frac{P_2}{P_1} = \checkmark$$

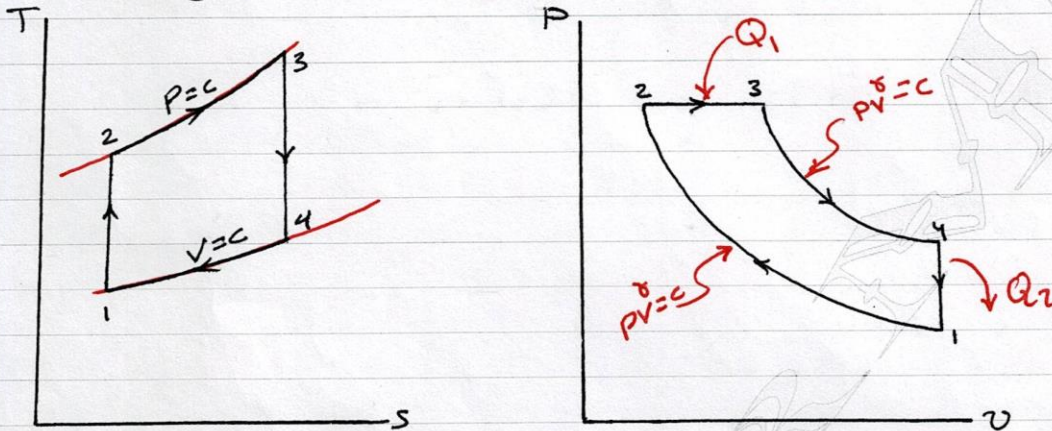
$$\frac{P_2 V_2^{\gamma}}{T_2} = \frac{P_3 V_3^{\gamma}}{T_3}$$

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \quad \Rightarrow \quad \frac{P_3}{P_2} = \checkmark$$

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## Diesel cycle

This is an important cycle on which all the diesel engines work. It is also known as constant pressure cycle as heat is received at a constant pressure.



**Process 1→2**, Isentropic compression

$$Q = W + \Delta U, \quad Q = 0, \quad m = 1 \text{ kg}$$

$$W = -\Delta U = -C_v(T_2 - T_1) = C_v(T_1 - T_2)$$

for positive value

$$W_c = C_v(T_2 - T_1)$$

**Process 2→3**, Reversible constant pressure heating

$$Q = W + \Delta U \\ = C_p(T_3 - T_2)$$

**Process 3→4**, Isentropic expansion

$$Q = W + \Delta U, \quad Q = 0$$

$$W = -\Delta U = -C_v(T_4 - T_3)$$

for positive value

$$W = C_v(T_3 - T_4)$$

Process 4 → 1, Reversible constant volume cooling

$$Q = W + \Delta U, \quad W = 0$$

$$Q = \Delta U = C_v(T_1 - T_4)$$

for positive value

$$Q = C_v(T_4 - T_1)$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2}$$

Process 2 → 3  $p = c$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow T_3 = T_2 \cdot \frac{V_3}{V_2} \quad \text{--- ①}$$

Process 3 → 4 isentropic

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3 \cdot \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$\because V_4 = V_1 \Rightarrow T_4 = T_3 \cdot \left(\frac{V_3}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_2}\right)^{\gamma-1} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \text{--- ②}$$

Sub. eq ① in ②

$$T_4 = T_2 \cdot \left(\frac{V_3}{V_2}\right) \cdot \left(\frac{V_3}{V_2}\right)^{\gamma-1} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_2 \left(\frac{V_3}{V_2}\right)^{\gamma} \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \Rightarrow T_1 = T_2 \cdot \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

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$$\eta_{th} = 1 - \frac{T_2 \cdot \left(\frac{V_3}{V_2}\right)^\gamma \cdot \left(\frac{V_1}{V_2}\right) - T_1 \left(\frac{V_1}{V_2}\right)}{\gamma (T_2 \cdot \left(\frac{V_3}{V_2}\right) - T_1)}$$

let  $r_v = \frac{V_1}{V_2}$  (compression ratio)

$\beta = \frac{V_3}{V_2}$  (cut off ratio)

$$\eta_{th} = 1 - \frac{T_2 \left( \beta^\gamma \frac{1}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}} \right)}{\gamma T_2 (\beta - 1)}$$

$$\eta_{th} = 1 - \frac{\beta^\gamma \frac{1}{r_v^{\gamma-1}} - \frac{1}{r_v^{\gamma-1}}}{\gamma (\beta - 1)}$$

$$\eta_{th} = 1 - \frac{(\beta^\gamma - 1)}{(\beta - 1) \cdot \gamma \cdot r_v^{\gamma-1}}$$

**Ex:** A diesel engine has an inlet temperature and pressure of 15°C and 1 bar respectively. The compression ratio is 12:1 and the maximum cycle temperature is 1100°C. Calculate the air standard thermal efficiency based on the Diesel Cycle.

**Sol:**

$$T_1 = 15 + 273 = 288 \text{ K}$$

$$T_3 = 1100 + 273 = 1373 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (r_v)^{\gamma-1} \quad \gamma = 1.4$$

$$T_2 = 778 \text{ K}$$

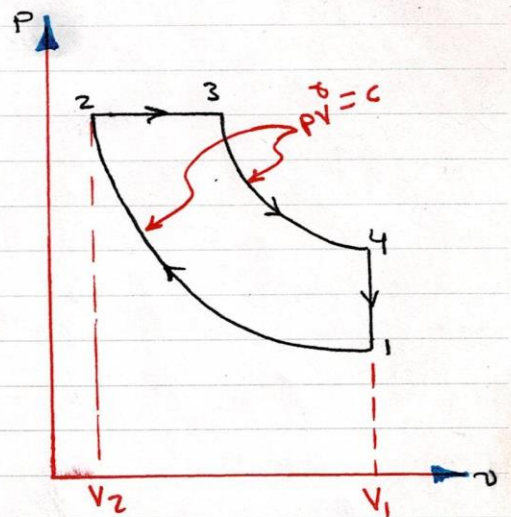
Process 2 → 3  $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$  ,  $P_2 = P_3$

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1373}{728} = 1.765$$

$$\frac{V_4}{V_3} = \frac{V_4}{V_2} \cdot \frac{V_2}{V_3} = \frac{V_1}{V_2} \cdot \frac{1}{V_3/V_2}$$

$$= 12 \cdot \frac{1}{1.765} = 6.8$$

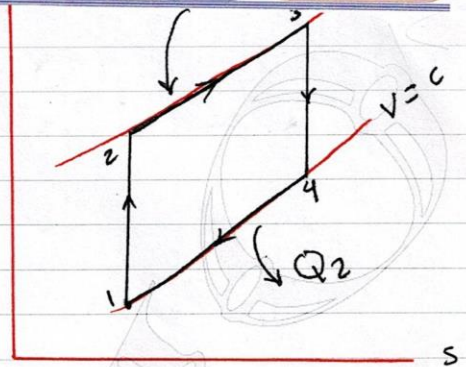
$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \Rightarrow \frac{1373}{T_4} = (6.8)^{0.4} \Rightarrow T_4 = 638 \text{ K}$$



$$Q_1 = C_p (T_3 - T_2) = 1.005 (1773 - 778) = 598 \text{ kJ/kg}$$

$$Q_2 = C_v (T_4 - T_1) = 0.718 (638 - 288) = 251 \text{ kJ/kg}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{251}{598} = 0.58 = 58\%$$



**Ex:** An engine uses air as a working substance, at the beginning of compression the pressure is 90 kPa and the temperature is 40°C. During the adiabatic compression the volume is reduced to one-sixteenth of its volume at the beginning of compression stroke. Heat is then added at constant pressure until the temperature is 1400°C. The stroke is completed by adiabatic expansion until the initial volume is reached, and the cycle is closed by constant volume process. Find: **a.** the temp. and at all four corner. **b.** thermal efficiency.

**Sol:**

\* Point -1-

$$P_1 = 90 \text{ kPa} \rightarrow T_1 = 313 \text{ K}$$

\* Point -2-

$$V_2 = \frac{1}{16} V_1$$

$$P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot P_1 = (16)^{1.4} \cdot 90 = 4365 \text{ kPa}$$

$$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \cdot T_1 = (16)^{0.4} \cdot 313 = 949 \text{ K}$$

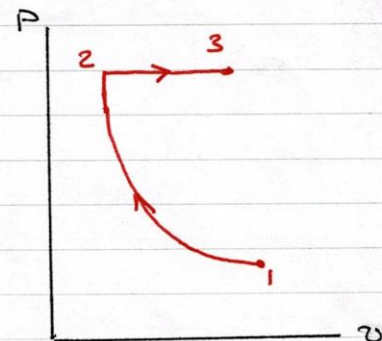
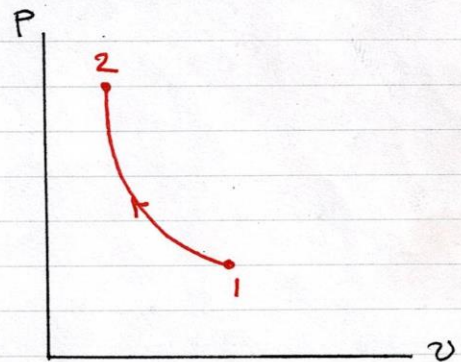
\* Point -3-

$$P_2 = P_3 = 4365 \text{ kPa}$$

$$T_3 = 1400 + 273 = 1673 \text{ K}$$

\* Point -4-

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_3}{V_1}\right)^{\gamma-1}$$



$$\frac{T_4}{T_3} = \left( \frac{V_3}{V_2} \cdot \frac{V_2}{V_1} \right)^{\gamma-1} = \left( \frac{T_3}{T_2} \cdot \frac{V_2}{V_1} \right)^{\gamma-1}$$

$$T_4 = T_3 \cdot \left( \frac{T_3}{T_2} \cdot \frac{V_2}{V_1} \right)^{\gamma-1}$$

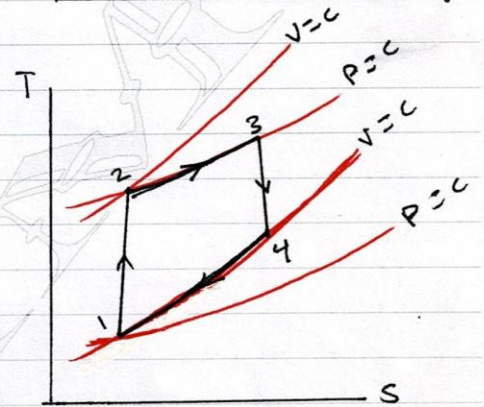
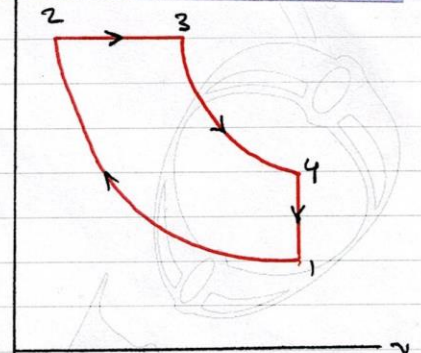
$$T_4 = 1673 \cdot \left( \frac{1673}{949} \cdot \frac{1}{16} \right)^{1.4} = 692 \text{ K}$$

$$P_4 = P_3 \left( \frac{T_4}{T_3} \right)^{\frac{\gamma}{\gamma-1}} = 4365 \left( \frac{692}{1673} \right)^{\frac{1.4}{0.4}}$$

$$= 198.6 \text{ kPa}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \rightarrow$$

$$\eta_{th} = 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{692 - 313}{1.4(1673 - 949)} = 0.63 = 63\%$$



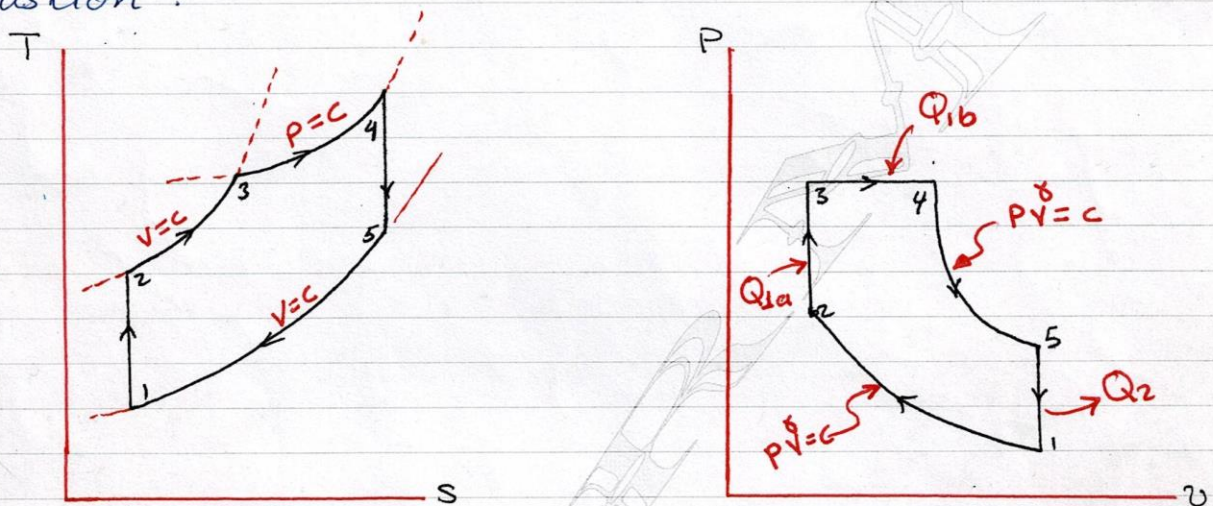
H.W: find the  $M_{net}$  from the shaded area on (P-v) diagram.

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## Dual combustion cycle

Modern oil engines although still called diesel engines are more closely derived from an engine invented by Ackroy-Stuart. The heat is supplied in two parts, the first part at constant volume and the remainder at constant pressure, hence the name "dual combustion".



**Process 1 → 2** Adiabatic compression  $PV^\delta = C$

$$Q = W + \Delta U, \quad Q = 0, \quad m = 1 \text{ Kg}$$

$$W = -\Delta U = -C_v(T_2 - T_1)$$

for positive value,  $W = C_v(T_2 - T_1)$

**Process 2 → 3** Reversible constant volume heating ( $Q_{1a}$ )

$$U = c \Rightarrow W = 0$$

$$Q_{1a} = C_v(T_3 - T_2)$$

**Process 3 → 4** Reversible constant pressure heating ( $Q_{1b}$ )

$$P = C$$

$$Q = W + \Delta U$$

$$Q_{1b} = h_4 - h_3$$

$$Q_{1b} = C_p(T_4 - T_3)$$

**Process 4 → 5** Adiabatic expansion  $PV^\delta = C$

$$Q = W + \Delta U, \quad Q = 0$$

$$W = -\Delta U = -C_v(T_5 - T_4)$$

for positive value,  $W = C_v(T_4 - T_5)$

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Process  $\rightarrow$  1

$$V = C \Rightarrow W = 0$$

$$Q = W + \Delta U$$

$$Q_2 = \Delta U = C_v (T_1 - T_5)$$

for positive value

$$Q_2 = C_v (T_5 - T_1)$$

$$* r_v = \frac{V_1}{V_2} = \text{Compression ratio}$$

$$* \alpha = \frac{P_3}{P_2} = \text{ratio of pressure during heat adding at constant volume.}$$

$$* \beta = \frac{V_4}{V_3} = \text{ratio of volume during heat adding at constant pressure.}$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{Q_2}{Q_{1a} + Q_{1b}}$$

$$\eta_{th} = 1 - \frac{C_v (T_5 - T_1)}{C_v (T_3 - T_2) + C_p (T_4 - T_3)} = 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma (T_4 - T_3)}$$

$$\eta_{th} = 1 - \frac{(\alpha \beta^\gamma - 1)}{[\alpha - 1 + \gamma \alpha (\beta - 1)] r_v^{\gamma - 1}}$$

if  $\alpha = 1$  (heat added only at  $p = c$ )

$$\eta_{th} = 1 - \frac{\beta^\gamma - 1}{r_v^{\gamma - 1} \cdot \gamma \cdot (\beta - 1)} \quad *$$

if  $\beta = 1$  (heat added only at  $v = c$ )

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma - 1}} \quad *$$

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**Ex:** An oil engine takes at 1.01 bar, 20°C and the maximum pressure is 69 bar. The compression ratio is 18. Calculate the air standard thermal efficiency based on the dual combustion cycle. Assuming that the heat added at constant volume is equal to the heat added at constant pressure.

**Sol:**

$$\eta_{th} = 1 - \frac{Q_2}{Q_{1a} + Q_{1b}}$$

$$= 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

Point (1)  $P_1 = 1.01 \text{ bar}$ ,  $T_1 = 298 \text{ K}$

Point (2)  $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 298(18)^{1.4-1} = 931 \text{ K}$

$$P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot P_1 = 57.8 \text{ bar}$$

Point (3)  $P_3 = 69 \text{ bar}$ ,  $\frac{P_2 V_2^{\gamma}}{T_2} = \frac{P_3 V_3^{\gamma}}{T_3}$ ,  $v = c$

$$T_3 = \frac{P_3}{P_2} \cdot T_2 = \frac{69}{57.8} \cdot 931 = 1112 \text{ K}$$

Point (4)  $P_4 = P_3 = 69 \text{ bar}$ ,  $Q_{1a} = Q_{1b}$

$$C_v(T_3 - T_2) = C_p(T_4 - T_3)$$

$$T_4 = \frac{1}{\gamma}(T_3 - T_2) + T_3 = 1241 \text{ K}$$

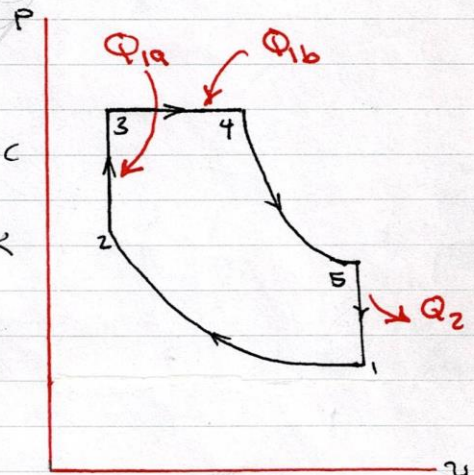
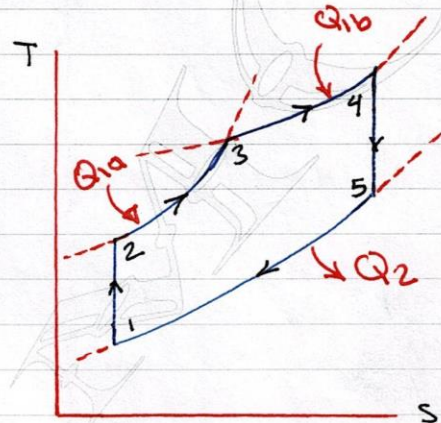
Point (5)  $\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_3} \cdot \frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$$

$$= \left(\frac{T_4}{T_3} \cdot 1 \cdot \frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1241}{1112} \cdot 1 \cdot \frac{1}{18}\right)^{0.4}$$

$$\therefore T_5 = 408 \text{ K}$$

$$\eta_{th} = 1 - \frac{408 - 293}{(1112 - 931) + 1.4(1241 - 1112)} = 0.681$$



$$= 68.1\%$$

## \* Mean effective pressure :

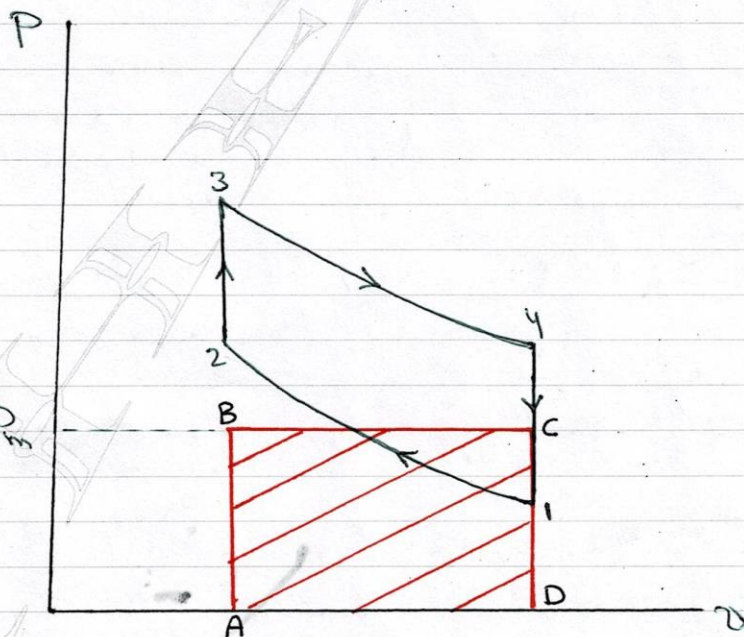
In internal-combustion engines the term work ratio is not such a useful concept, since the work done on and by the working fluid takes place inside one cylinder.

In order to compare reciprocating engines another term is defined called the mean effective pressure. The mean effective pressure is defined as the height of a rectangle having the same length and area as the cycle plotted on a  $P$ - $v$  diagram.

for Otto cycle, the rectangle ABCDA is the same length as the cycle 12341, and area ABCDA is equal to area 12341, then the mean effective pressure,  $P_m$  is the height AB of the rectangle.

The work done per kg of air can therefore be written as

$$W = \text{area ABCDA} = P_m (v_1 - v_2)$$



Ex: Calculate the thermal efficiency of an air standard Diesel Cycle with a compression ratio 15/1 and max. & min. cycle temperature of 1650°C & 15°C respectively. The maximum pressure is 45 bar.

Sol:

$$\frac{V_1}{V_2} = 15$$

$$\eta_{th} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$T_2 = T_1 \cdot \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow T_2 = 577.8^\circ\text{C}$$

$$\frac{V_3}{V_2} = \frac{T_3}{T_2} \Rightarrow \frac{V_3}{V_2} = 2.26$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = \left(\frac{V_3}{V_1}\right)^{\gamma-1} \quad (V_4 = V_1)$$

$$T_4 = T_3 \cdot \left(\frac{V_3}{V_1} \cdot \frac{V_2}{V_2}\right)^{\gamma-1} = T_3 \cdot \left(\frac{V_3}{V_2} \cdot \frac{V_2}{V_1}\right)^{\gamma-1}$$

$$T_4 = 628.9^\circ\text{C}$$

$$\eta_{th} = 1 - \frac{901.9 - 288}{1.4(1923 - 850.8)}$$

$$= 0.59$$

$$= 59\%$$

$$Q_1 = C_p(T_3 - T_2) \Rightarrow Q_1 = 1077.36 \frac{\text{kJ}}{\text{kg}}$$

$$W = \eta_{th} \cdot Q_1 \Rightarrow W = 636.6 \text{ kJ/kg}$$

$$V_1 = \frac{mRT_1}{P_1} \Rightarrow V_1 = 0.8319 \text{ m}^3 \text{ \& } V_2 = \frac{V_1}{15} = 0.0592 \text{ m}^3$$

$$W = P_m(V_1 - V_2)$$

$$636.6 = P_m(0.8319 - 0.0592)$$

$$\therefore P_m = 830 \text{ kPa}$$



## Sheet No. -7-

\* Sketch the processes on T-s & P-v diagram.

Q1:

What is the highest thermal efficiency possible for a heat engine operating between  $800^{\circ}\text{C}$  and  $15^{\circ}\text{C}$ .

Ans -  $73.2\%$

Q2:

In a Carnot cycle operating between  $307^{\circ}\text{C}$  and  $17^{\circ}\text{C}$  the max. and min. pressure are  $62.4\text{ bar}$  and  $1.04\text{ bar}$ . Calculate the thermal efficiency and the work ratio.

Ans -  $50\%$ ,  $0.287$

Q3:

A closed cycle gas turbine unit operating with maximum and minimum temperature of  $760^{\circ}\text{C}$  and  $20^{\circ}\text{C}$  has a pressure ratio of  $7/1$ . Calculate the ideal thermal efficiency and the work ratio.

Ans -  $42.7\%$ ,  $0.503$

Q4:

In an air standard Otto cycle the maximum and minimum temperatures are  $1400^{\circ}\text{C}$  and  $15^{\circ}\text{C}$ . The heat supplied per kg of air is  $800\text{ kJ}$  calculate the compression ratio and thermal efficiency. Calculate also the ratio of maximum to minimum pressure in the cycle.

Ans -  $5.26/1$ ,  $48.6\%$ ,  $30.5/1$

Q5:

A four-cylinder petrol engine has a swept volume of  $2000\text{ cm}^3$  and the clearance volume in each cylinder is  $60\text{ cm}^3$ . Calculate the air standard thermal efficiency. If the induction conditions are  $1\text{ bar}$  and  $24^{\circ}\text{C}$ , and the maximum cycle temperature is  $1400^{\circ}\text{C}$ , calculate the mean effective pressure based on the air standard cycle.

Ans -  $59\%$ ,  $5.27\text{ bar}$

Q6:

In a dual combustion cycle the maximum temperature is  $2000^{\circ}\text{C}$  and the maximum pressure is  $70\text{ bar}$ . Calculate the thermal efficiency and the mean effective pressure when the pressure and temperature at the start of compression are  $1\text{ bar}$  and  $17^{\circ}\text{C}$  respectively. The compression ratio is  $18/1$ .

Ans -  $63.6\%$ ,  $10.5\text{ bar}$



Q7:

An air standard dual combustion cycle has a mean effective pressure of 10 bar. The minimum pressure and temperature are 1 bar and  $17^{\circ}\text{C}$  respectively, and the compression ratio is 16:1. Calculate the maximum cycle temperature when the thermal efficiency is 60%. The maximum cycle pressure is 60 bar.

Ans -  $1959^{\circ}\text{C}$

Q8:

A gasoline engine has a volumetric compression ratio of 9:1. The state before compression is 290 K, 90 kPa and the peak cycle temperature is 1800 K. Find the pressure after expansion, the cycle net work and the cycle efficiency.

Ans - 231.9 kPa, 58.4%, 461 kJ/kg

Q9:

In an ideal diesel cycle the compression ratio is 15:1 and the expansion ratio is 7.5:1. The pressure and temperature at the beginning of compression are 98 kPa and  $44^{\circ}\text{C}$ , the pressure at the end of expansion is 288 kPa. Find: a. the maximum temperature b. the thermal efficiency.

Ans -  $1597^{\circ}\text{C}$ , 60.5%

اكتلم يميني قلوب ليشين كما  
تيا اكلار اذا ما عسر الطر  
و اعلم بجلي يعني عن قلب صاحبه  
كاتبلي حواد نظمة بقر

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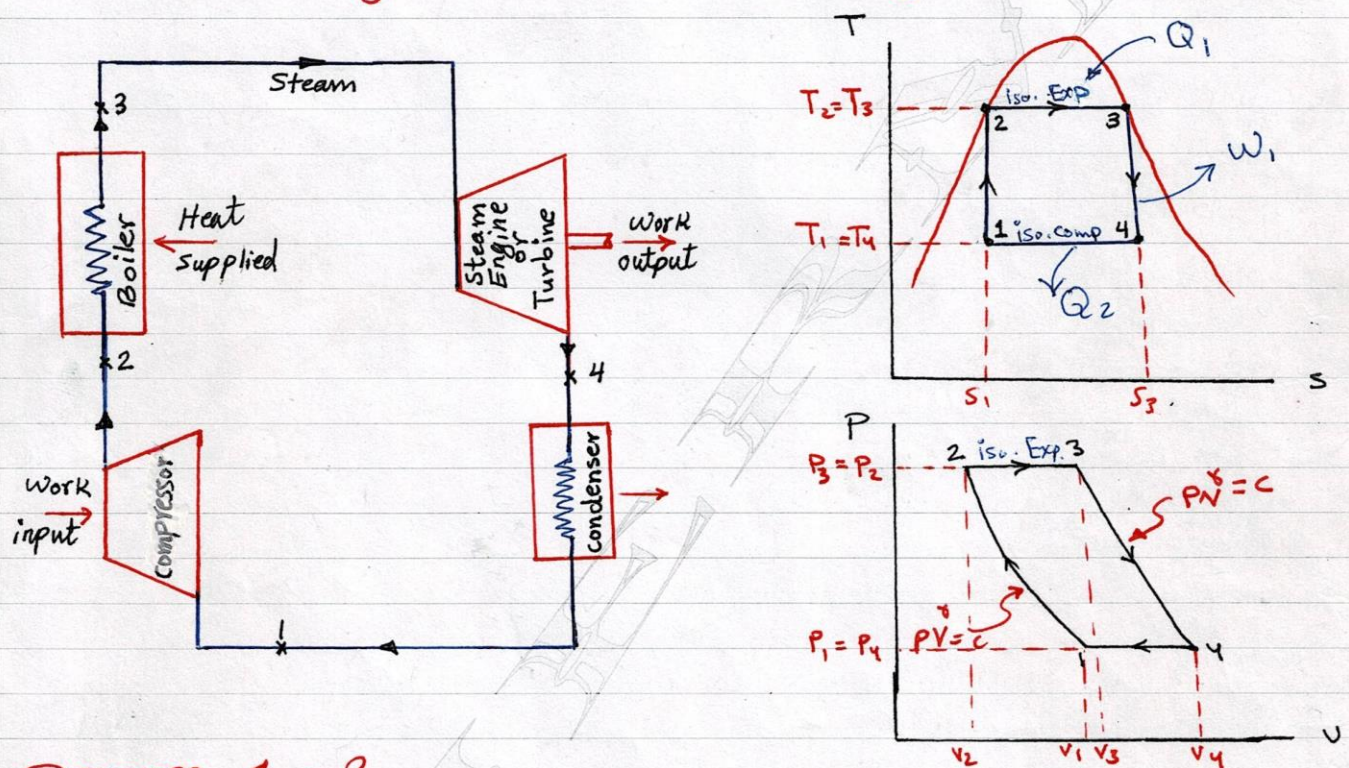
CHAPTER SIX

The Steam Cycles

## The Steam Cycles

In a vapour cycle, all the theory of air cycles remains the same, except the working substance, which is steam. The steam may be in any form, wet, dry saturated or superheated.

### \* Carnot Cycle with steam as working substance :



#### Process 1 → 2

Isentropic compression of wet steam in compressor; the pressure and temperature rises, and no heat is absorbed or rejected during this process.  $Q=0$

#### Process 2 → 3

The saturated water at point 2, is isothermally converted into dry saturated steam, in a boiler, and the heat is absorbed at constant temperature.

$$q_{2 \rightarrow 3} = \text{change in entropy} \times \text{Absolute temperature}$$

$$q_{2 \rightarrow 3} = (S_3 - S_2) \cdot T_2 = (S_3 - S_2) \cdot T_3$$

## Process 3 → 4

The dry steam now expands isentropically in a steam engine or turbine. The pressure and temperature falls from  $P_3$  to  $P_4$  and  $T_3$  to  $T_4$  respectively, and no heat is supplied or rejected during this process.

## Process 4 → 1

The wet steam is now isothermally condensed in condenser and the heat is rejected at constant temperature and pressure.

$$q_{4 \rightarrow 1} = (S_4 - S_1) \cdot T_1 = (S_4 - S_1) \cdot T_4$$

$$\sum Q = \sum W$$

∴ Work done during the cycle = Heat absorbed - Heat rejected

$$W.D = Q_1 - Q_2 \quad , \text{ for 1 kg}$$

$$W.D = q_{2 \rightarrow 3} - q_{4 \rightarrow 1}$$

$$W.D = (S_3 - S_2)T_2 - (S_4 - S_1)T_1$$

$$= (S_3 - S_2)(T_2 - T_1) \quad , \quad S_3 = S_4 \quad , \quad S_2 = S_1$$

or

$$W.D = (S_4 - S_1)(T_2 - T_1) \quad , \quad T_2 = T_3 \quad , \quad T_1 = T_4$$

or

$$W.D = (S_4 - S_1)(T_3 - T_4)$$

or

$$W.D = (S_3 - S_2)(T_3 - T_4)$$

$$\eta = \frac{\text{work done}}{\text{heat added}} = \frac{(S_3 - S_2)(T_2 - T_1)}{(S_3 - S_2)T_2} = \frac{T_2 - T_1}{T_2}$$

$$\eta_{th} = 1 - \frac{T_1}{T_2}$$

## Notes:

1. Since the heat absorbed is at the highest temperature and rejected at the lowest temperature, the Carnot cycle would give a maximum possible efficiency.
2. It may be noted that it is impossible to make a steam engine working on Carnot cycle, the isothermal expansion  $2 \rightarrow 3$  and the isothermal compression  $4 \rightarrow 1$  will have to be carried out extremely slow to ensure that the steam is always at constant temperature. But the isentropic expansion  $3 \rightarrow 4$  and isentropic compression  $1 \rightarrow 2$  should be carried out as quickly as possible in order to approach ideal isentropic condition. We know that sudden changes in the speed of an engine are not possible in actual practice, therefore it is impossible to realise Carnot's engine in actual practice.
3. At state 1 the steam is wet at  $T_1$ , it is difficult to stop condensation at the point 1 and then compress it just to state 2. It is more convenient to allow the condensation process to proceed to completion.
4. The working fluid is water at the new state, and this can be conveniently pumped to boiler pressure, the pump has much smaller dimensions than it would have if it had to pump a wet vapour, the compression process is carried out more efficiently, and the equipment required is simpler and less expensive.

**Ex:** A power plant is supplied with dry saturated steam at a pressure of 16 bar and exhausts at 0.2 bar. find the efficiency of the Carnot cycle.

**Sol:** from Steam tables, at 16 bar,  $T_s = 201.4 + 273 = 474.4 \text{ K} = T_2$   
from Steam tables, at 0.2 bar,  $T_s = 60.1 + 273 = 333.1 \text{ K} = T_1$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{333.1}{474.4} = 0.298 \text{ or } 29.8\%$$



**Ex:** In a Carnot cycle, heat is supplied at  $350^{\circ}\text{C}$  and is rejected at  $25^{\circ}\text{C}$ . The working fluid is water, which while receiving heat, evaporates from liquid at  $350^{\circ}\text{C}$  to steam at  $350^{\circ}\text{C}$ . From steam tables the entropy change for this process is  $1.438 \text{ kJ/kg}\cdot\text{K}$ .

If the cycle operates on a stationary mass of  $1 \text{ kg}$  of water, find the heat supplied, work done and heat rejected per cycle. What is the pressure of water during heat reception.

**Sol:** the heat supplied  $= (S_3 - S_2) T_2$        $T_2 = 350 + 273$   
 $= 1.438 \times 623$        $= 623 \text{ K}$   
 $= 895.871 \text{ kJ/kg}$

the work done  $= (S_3 - S_2) (T_2 - T_1)$        $T_1 = 25 + 273$   
 $= 1.438 \times (623 - 298)$        $= 298 \text{ K}$   
 $= 467.351 \text{ kJ/kg}$

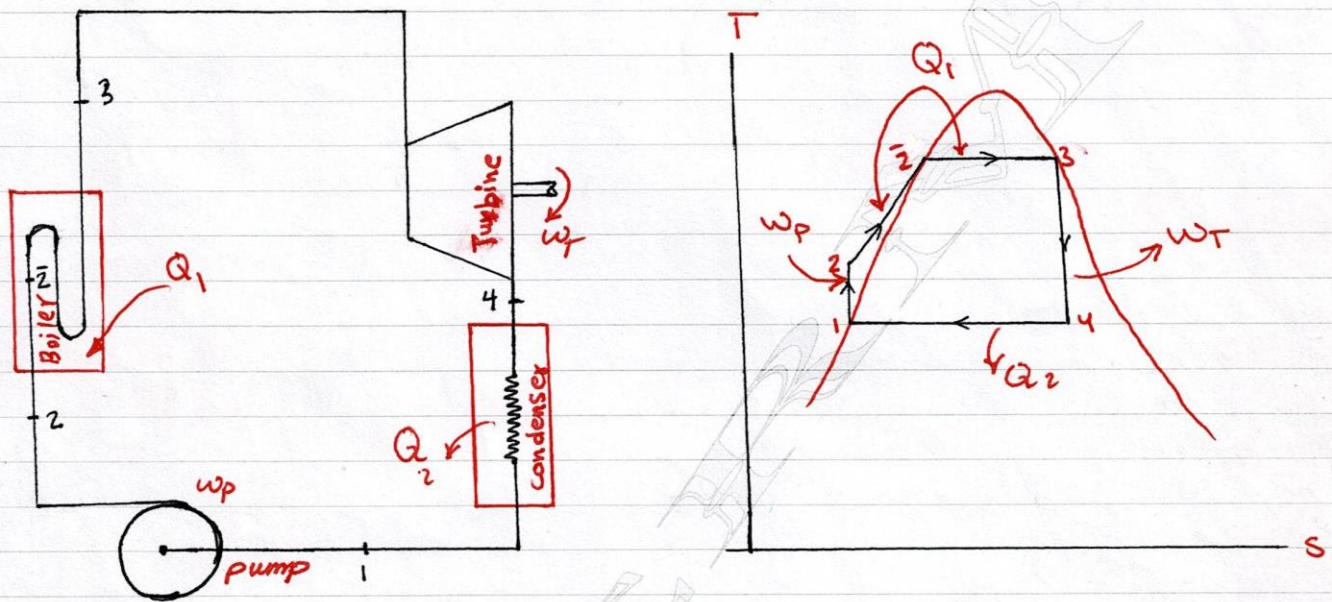
the heat rejected  $= (S_4 - S_1) \cdot T_1$        $S_4 - S_1 = S_3 - S_2$   
 $= 1.438 \times 298$   
 $= 428.521 \text{ kJ/kg}$

from steam tables at  $T_s = 350^{\circ}\text{C}$  ,  $P = 165.35 \text{ bar}$

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## \* Rankine Cycle :

The Rankine cycle is an ideal cycle for comparing the performance of Steam plants. It is modified form of Carnot Cycle, in which the condensation process (4→1) is continued until the steam is condensed into water.



- By the flow Energy Equation ( $\Delta Z \approx 0$ ,  $\Delta C \approx 0$ )

1. Boiler 2→3 (Heat added,  $Q_1$ )

$$h_2 + q = h_3 + w, \quad w = 0 \text{ in the boiler}$$

$$\therefore q_{2 \rightarrow 3} = h_3 - h_2 \Rightarrow \boxed{Q_{2 \rightarrow 3} = m(h_3 - h_2)}$$

2. Turbine 3→4 (work output,  $W_T$ )

$$h_3 + q = h_4 + w, \quad q = 0, \text{ isentropic Exp.}$$

$$w_{3 \rightarrow 4} = h_3 - h_4 \Rightarrow \boxed{W_{3 \rightarrow 4} = m(h_3 - h_4)}$$



3. Condenser  $4 \rightarrow 1$  (Heat rejected,  $Q_2$ )

$$h_4 + q = h_1 + w, \quad w = 0$$

$$q_{4 \rightarrow 1} = h_1 - h_4, \text{ for positive value, } \boxed{Q_{4 \rightarrow 1} = m(h_4 - h_1)}$$

4. Pump  $1 \rightarrow 2$  (work input,  $w_p$ )

$$h_1 + q = h_2 + w, \quad q = 0$$

$$w_{1 \rightarrow 2} = h_1 - h_2, \text{ for positive value, } \boxed{W_{1 \rightarrow 2} = m(h_2 - h_1)}$$

$$\text{or } W_{\text{pump}} = v_f (P_2 - P_1)$$

$$\begin{aligned} \text{Net work done} &= W_{3 \rightarrow 4} - W_{1 \rightarrow 2} \\ &= W_T - W_C \end{aligned}$$

$$W.D = (h_3 - h_4) - (h_2 - h_1)$$

if the feed pump work is neglected,  $w_p = 0$

$$W.D = h_3 - h_4$$

$$\text{Heat added} = q_1 = h_3 - h_2$$

$$\text{Rankine efficiency, } \eta_R = \frac{W.D}{Q_{\text{add.}}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

$$\text{If } w_p = 0 \quad \eta_R = h_3 - h_4 / h_3 - h_1$$

\* **Efficiency ratio:** It is also known as relative efficiency. It is defined as the ratio of thermal efficiency (or actual cycle efficiency) to Rankine efficiency (or ideal cycle efficiency). Mathematically,

$$\boxed{\text{Efficiency ratio} = \frac{\text{Thermal efficiency}}{\text{Rankine efficiency}}}$$

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\* **Work ratio** : It is defined as the ratio of net work output to the gross (engine or turbine) output.

$$\text{Work ratio} = \frac{\text{Net work output}}{\text{Gross output}} = \frac{\text{Turbine work} - \text{Compressor work}}{\text{Turbine work}}$$

It may be noted that the Carnot cycle, despite of its high ideal thermal efficiency, has low work ratio. It is one of the reasons that Carnot cycle is not attempted. The higher value of work ratio also means a smaller size of the plant.

\* **Specific Steam Consumption (S.S.C.)** :

It is also known as steam rate or specific rate of flow of steam. It is defined as the mass of steam that must be supplied to a steam engine or turbine in order to develop a unit amount of work or power output. The amount of work or power output is usually expressed in kilowatt hour (kWh).

$$\text{S.S.C.} = \frac{1 \text{ kWh}}{W} = \frac{3600}{W} = \frac{3600}{h_3 - h_4} \quad \text{kg/kWh} \cdot h \quad , W_p = 0$$

W = net work done

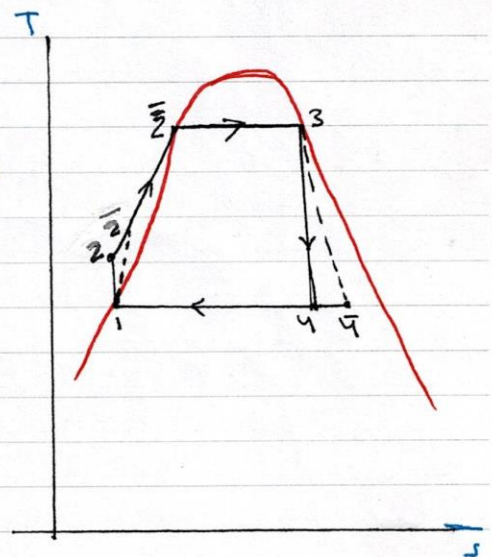
\* **Isentropic efficiency** :

a. For expansion process: (Turbine, 3 → 4)

$$\eta_{\text{ise.}} = \frac{\text{actual W.D}}{\text{Isentropic W.D}} = \frac{h_3 - h_4}{h_3 - h_4}$$

b. For compression process: (Pump)

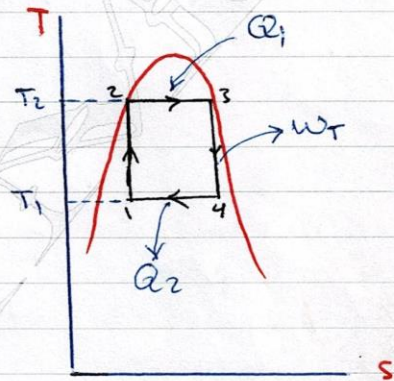
$$\eta_{\text{ise.}} = \frac{\text{Isentropic W.D}}{\text{actual W.D}} = \frac{h_2 - h_1}{h_2 - h_1}$$



**Ex:** A steam power plant operates between a boiler pressure of 42 bar and a condenser pressure of 0.035 bar. Calculate for these limits the cycle efficiency, the work ratio, and the specific steam consumption:

- for a Carnot cycle using wet steam,
- for a Rankine cycle with dry saturated steam at entry to the turbine,
- for the Rankine cycle of (b), when the expansion process has an isentropic efficiency of 80%

**Sol:** from steam tables  
at  $p = 42 \text{ bar}$ ,  $T_s = 253.2^\circ\text{C}$   
at  $p = 0.035 \text{ bar}$ ,  $T_s = 26.7^\circ\text{C}$



a

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}} = 1 - \frac{26.7 + 273}{253.2 + 273} = 0.432 \text{ or } 43.2\%$$

$$Q_1 = h_3 - h_2 = h_{g3} - h_{f2} \quad , \text{ at } p = 42 \text{ bar}$$

$$= 2800 - 1105$$

$$= 1695 \text{ kJ/kg}$$

$$\text{Net work done} = \eta_{\text{Carnot}} \times Q_1$$

$$= 0.432 \times 1695$$

$$= 732.3 \text{ kJ/kg}$$

$$W_T = h_3 - h_4 \quad , \quad s_3 = s_4 = s_g \text{ at } p = 42 \text{ bar}$$

$$= 6.05 \text{ kJ/kg}$$

$$s_4 = s_f + x_4 s_{fg} \quad , \text{ at } p = 0.035 \text{ bar}$$

$$6.05 = 0.391 + x_4 \times 8.13 \Rightarrow x_4 = 0.696$$

$$h_4 = h_f + x_4 h_{fg} \quad , \text{ at } p = 0.035 \text{ bar}$$

$$= 112 + 0.696 \times 2438 = 1808 \text{ kJ/kg}$$

$$W_T = h_3 - h_4 = 2800 - 1808 = 992 \text{ kJ/kg}$$

$$\text{Work ratio} = \frac{\text{net work}}{\text{gross work}} = \frac{732.3}{992} = 0.738$$

$$\text{S.S.C.} = 3600 / W_{\text{net}} = 3600 / 732.3 = 4.92 \text{ kg/kWh}$$

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**b.**

$$h_3 = 2800 \text{ kJ/kg}, \quad h_4 = 1808 \text{ kJ/kg}$$

$$h_2 = 1105 \text{ kJ/kg}, \quad h_1 = 111.9 \text{ kJ/kg}$$

$$v_1 = v_f = 0.001 \text{ m}^3/\text{kg}$$

$$\eta_R = \frac{W.D}{Q_1} = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)}$$

$$\text{Pump work} = h_2 - h_1 = v_1 (P_2 - P_1)$$

$$= 0.001 (42 - 0.035) \times 10^2$$

$$= 4.2 \text{ kJ/kg}$$

$$h_2 = h_1 + \text{Pump work} = 111.9 + 4.2$$

$$= 116.1 \text{ kJ/kg}$$

$$w_T = h_3 - h_4 = 2800 - 1808 = 992 \text{ kJ/kg}$$

$$w_p = 4.2 \text{ kJ/kg}$$

$$q_1 = h_3 - h_2 = 2800 - 116.1 = 2684 \text{ kJ/kg}$$

$$\text{work ratio} = \frac{w_{\text{net}}}{\text{gross work}} = \frac{992 - 4.2}{992} = 0.995$$

$$\text{S.S.C.} = \frac{3600}{w_{\text{net}}} = \frac{3600}{992 - 4.2} = 3.64 \text{ kg/kw.h}$$

$$\eta_R = \frac{W.D}{Q_1} = \frac{992 - 4.2}{2684} = 0.368 \text{ or } 36.8\%$$

**c.**

$$\eta_{\text{ise.}} = \frac{h_3 - h_4}{h_3 - h_4} = 0.8 \Rightarrow 0.8 (2800 - 1808) = 2800 - h_4$$

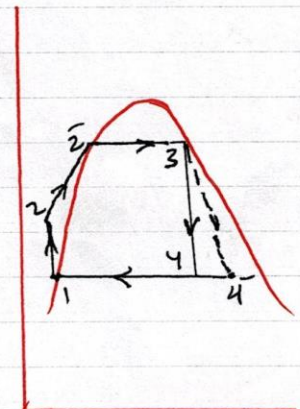
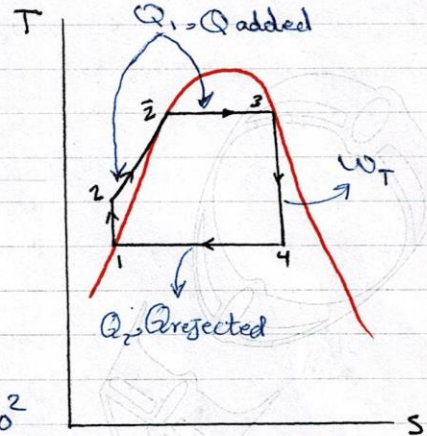
$$\Rightarrow h_4 = 2006.4 \text{ kJ/kg}$$

$$\eta_R = \frac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_2)} = \frac{(2800 - 2006.4) - 4.2}{2800 - 116.1}$$

$$= 0.294 \text{ or } 29.4\%$$

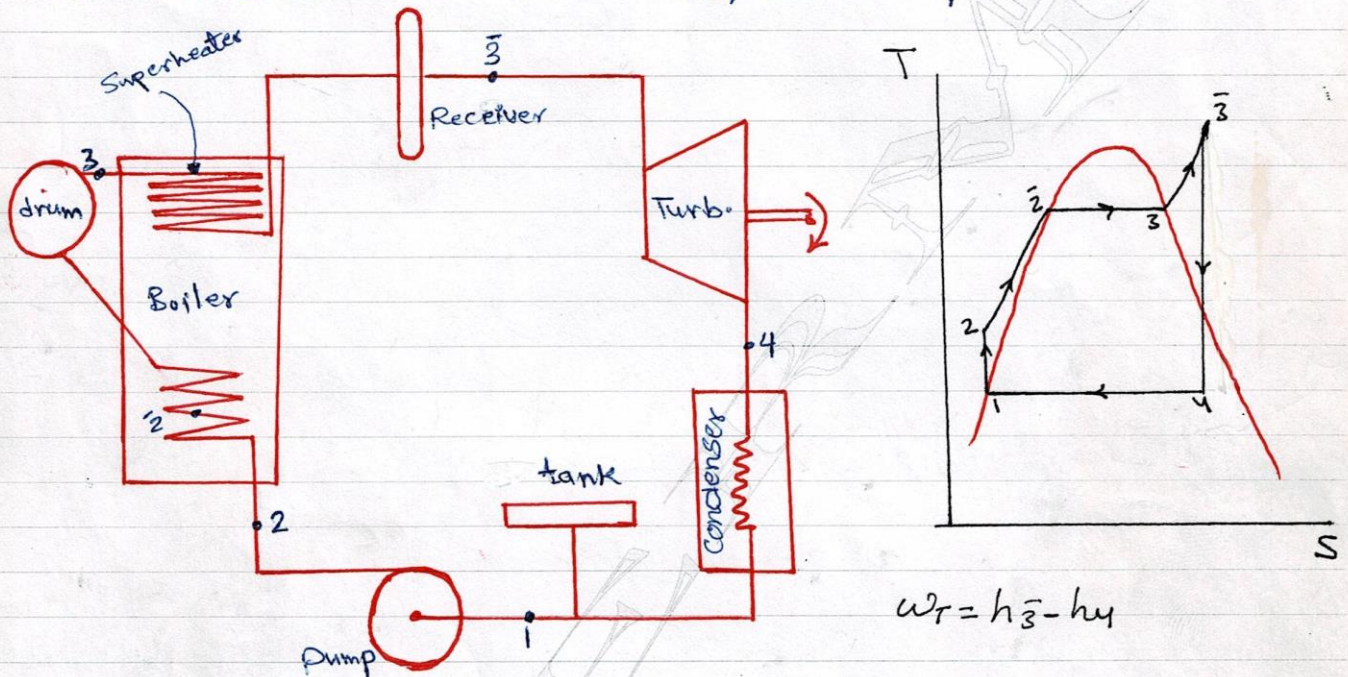
$$W.R. = \frac{(2800 - 2006.4) - 4.2}{2800 - 2006.4} = 0.9947$$

$$\text{S.S.C.} = \frac{3600}{793.6 - 4.2} = 4.56 \text{ kg/kw.h}$$



## \* Rankine Cycle with Superheated Steam:

The average temperature at which heat is supplied in the boiler can be increased by superheating the steam. Usually the dry saturated steam from the boiler drum is passed through a second bank of smaller bore tubes within the boiler. This bank is situated such that it is heated by hot gases from the furnace until the steam reaches the required temperature.



Assuming  $w_p = 0$  (always)

$$Q_1 = Q_{add} = h_{\bar{3}} - h_1 \quad , \quad W.D = W_T = h_{\bar{3}} - h_4$$

$$\eta_{th} = \frac{W.D}{Q_{add}} = \frac{h_{\bar{3}} - h_4}{h_{\bar{3}} - h_1}$$

**Ex:** A Steam turbine operating on the Rankine cycle received steam from the boiler at 3.5 Mpa and 350°C and exhaust to the condenser at 10 kpa. The condensate is then returned to the boiler by the feed pump. Calculate, neglecting all losses:

- the energy supplied in the boiler per kg of steam,
- the dryness fraction of steam entering the condenser,
- the Rankine efficiency.

**Sol:** At 10 kPa,  $h_1 = h_f = 192 \text{ kJ/kg}$   
 at 3.5 MPa and 350°C,  $h_3 = 3104 \text{ kJ/kg}$ ,  $S_3 = 6.6579 \text{ kJ/kg}\cdot\text{K}$   
 $S_4 = S_f + x_4 S_{fg}$ , at 10 kPa  
 $S_3 = S_4 \Rightarrow 6.657 = 0.6493 + x_4 \cdot 7.5 \Rightarrow x_4 = 0.8$

$$h_4 = h_f + x h_{fg} = 191.8 + 0.8 \times 2392.8, \text{ at } 10 \text{ kPa}$$

$$= 2106 \text{ kJ/kg}$$

Energy Supplied =  $h_3 - h_1$   
 $= 3104 - 192$   
 $= 2912.2 \text{ kJ/kg}$

$$M_R = \frac{h_3 - h_4}{h_3 - h_1} = \frac{3104 - 2106}{3104 - 191.8} = 0.3427 \text{ or } 34.27\%$$

**Ex:** Compare the Rankine cycle performance of example in page (211) with that obtained when the steam is superheated to 500°C, neglecting feed water work.

**Sol:** from Steam tables at 42 bar and 500°C

$$h_1 = 3442.6 \text{ kJ/kg}, S_1 = 7.066 \text{ kJ/kg}\cdot\text{K}$$

$$= S_2$$

$$S_2 = S_f + x S_{fg}$$

$$7.066 = 0.391 + x_2 \cdot 8.13 \Rightarrow x_2 = 0.821$$

$$h_2 = h_f + x_2 h_{fg} = 112 + 0.821 \times 2438$$

$$= 2113 \text{ kJ/kg}$$

$$h_3 = h_f = 112 \text{ kJ/kg}$$

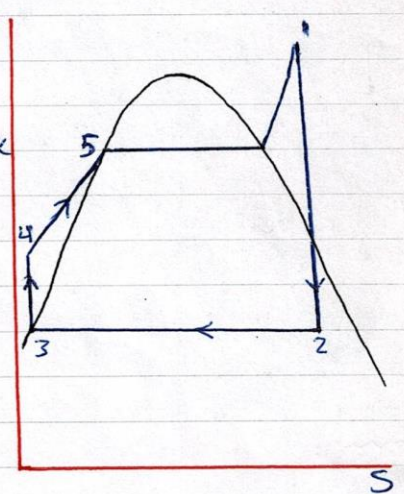
$$W_T = h_1 - h_2 = 3442.6 - 2113 = 1329.6 \text{ kJ/kg}$$

$$Q_1 = \text{heat Supplied} = h_1 - h_3 = 3442.6 - 112 = 3330.6 \text{ kJ/kg}$$

$$\eta_R = \frac{h_1 - h_2}{h_1 - h_3} = \frac{1329.6}{3330.6} = 0.399 \text{ or } 39.9\%$$

$$S.S.C = \frac{3600}{h_1 - h_2} = \frac{3600}{1329.6} = 2.71 \text{ kJ/kWh}$$

\* the thermal efficiency has increased due to superheating and the improvement in S.S.C is even more marked



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## \* Reheat Steam Cycle :

It is desirable to increase the average temperature at which heat is supplied to the steam, and also to keep the steam as dry as possible in the lower pressure stages of the turbine. The wetness at exhaust should be not greater than 10%. The high boiler pressures are required for high efficiency, but that expansion in one stage can result in exhaust steam which is wet. This is a condition which is improved by superheating the steam. The exhaust steam condition can be improved most effectively by reheating the steam, the expansion being carried out in two stages.

$$\text{Heat Supplied} = Q_{234} + Q_{56}$$

$$w_p \approx 0$$

$$Q_{234} = Q_{134} = h_4 - h_1$$

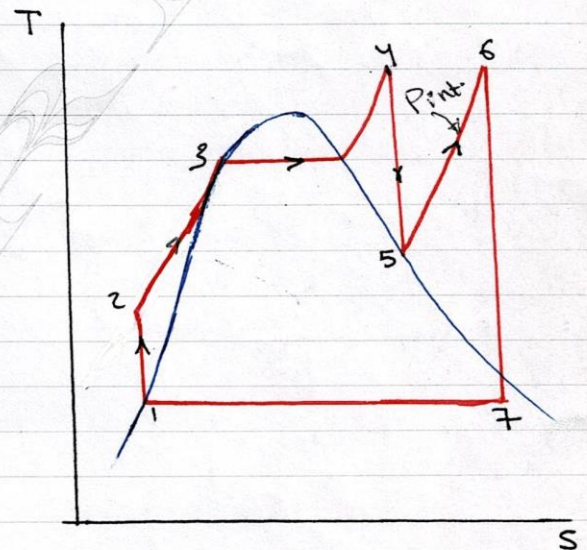
$$Q_{56} = h_6 - h_5$$

$$\text{Work output} = W_{45} + W_{67}$$

$$W_{45} = h_4 - h_5$$

$$W_{67} = h_6 - h_7$$

$$\eta_{th} = \frac{(h_4 - h_5) + (h_6 - h_7)}{(h_4 - h_1) + (h_6 - h_5)}$$



**Ex:** Calculate new cycle efficiency if reheat is included in the plant of above example. The steam condition at inlet to the turbine are 42 bar and 500°C and condenser pressure is 0.035 bar as before. Assume that the steam is just dry saturated on leaving the first turbine, and reheated to the initial temperature. Neglect feed water work.

**Sol:**  $h_1 = 3442.6 \text{ kJ/kg}$  ,  $s_1 = 7.066 \text{ kJ/kg} \cdot \text{K}$

$$s_1 = s_2 = s_g \Rightarrow P_2 = 2.4 \text{ bar} = P_{\text{intermediate}}$$





## Sheet No. -8-

**Q1: A.** Steam is supplied, dry saturated at 40 bar to a turbine and the condenser pressure is 0.035. If the plant operates on the Rankine Cycle, Calculate per kg of Steam: **a.** the work output neglecting feed pump work, **b.** the work required for feed pump **c.** the heat transferred to the condenser cooling water, and the amount of cooling water required through the condenser if the temperature rise of water is assumed to be 5.5 K, **d.** the heat supplied **e.** the Rankine efficiency  
**Ans.** [ 982.4 kJ, 4 kJ, 1706.6 kJ, 74.1 kg, 2685 kJ, 36.6% ]

**B.** for the same steam conditions calculate the efficiency for a Carnot cycle operating with wet steam.  
**Ans.** [ 43% ]

**Q2:** Repeat problem (1-A) for a steam supply condition of 40 bar and 350°C and the same condenser pressure.  
**Ans.** [ 1125 kJ, 4 kJ, 1857 kJ, 80.5 kg, 2978 kJ, 37.8% ]

**Q3:** Steam is supplied to a two-stage turbine at 40 bar and 350°C. It expands in the first turbine until it is just dry saturated, then it is reheated to 350°C and expanded through the second-stage turbine, the condenser pressure is 0.035 bar. Calculate the work output and the heat supplied per kg of steam for the plant, assuming ideal processes and neglecting the feed pump term, and calculate the cycle efficiency.  
**Ans.** [ 1290 kJ, 3362 kJ, 38.4% ]

**Q4:** If the expansion processes in the turbine of (Q3) have isentropic efficiencies of 84% and 78% respectively in the 1<sup>st</sup> and 2<sup>nd</sup> stages, calculate the work output and the heat supplied per kg of steam, the thermal efficiency and the specific steam consumption.  
1026 kJ, 3311 kJ, 31.1%, 3.51 kg/kWh

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CHAPTER SEVEN

Vapour Compression  
Refrigeration System

## „ Vapour Compression Refrigeration System „

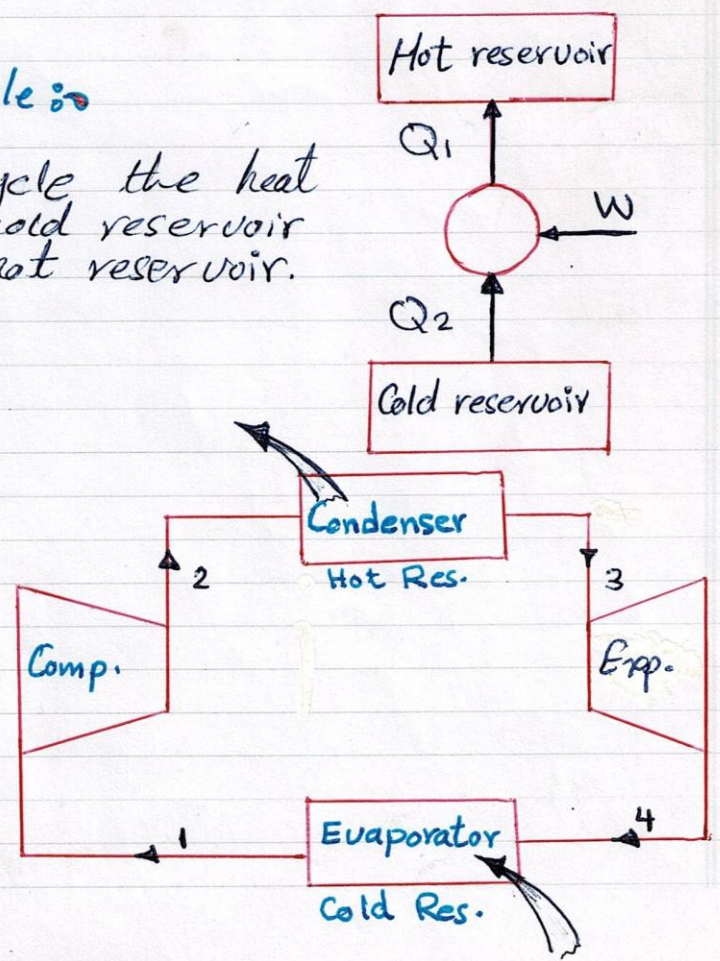
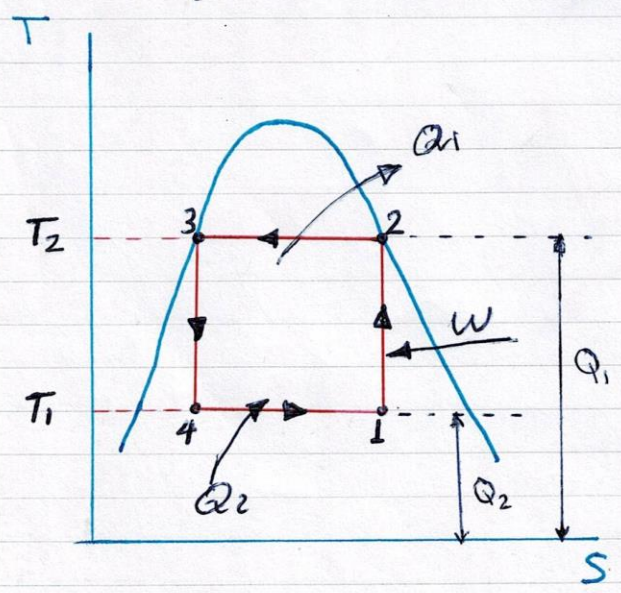
Refrigeration in the engineering sense, means maintaining a system at a temperature less than the temperature of the surroundings. This will not occur naturally, so a device must be developed that will maintain this condition.

The vapour compression refrigeration system is now used for all purpose refrigeration. It is generally used for all industrial purposes from a small domestic refrigerator to a big air conditioning plant.

A reversed Carnot engine will removed heat from a low temperature reservoir and deliver this energy, plus work necessary to transfer the heat, to high temperature reservoir. The refrigerated system in this case is the low temperature reservoir.

### \* Reversed Carnot Cycle :

In Refrigeration Carnot cycle the heat is absorbed from the cold reservoir and rejected to the hot reservoir.



## \* Processes of Refrigeration Carnot Cycle :

- 1 → 2 : Work input to the compressor to compress dry saturated vapour, rising its pressure and temperature to dry or superheated condition.
- 2 → 3 : heat rejected in condenser from the vapour to the ambient, changing the dry, or superheated vapour to saturated liquid.
- 3 → 4 : throttling the saturated liquid (expansion) changing it to wet vapour.
- 4 → 1 : heat absorbed from cold reservoir, changing the wet vapour of low quality to wet vapour of high quality.

$$\text{work input (process } 1 \rightarrow 2) = Q_1 - Q_2$$

$$\text{Heat rejected ( " } 2 \rightarrow 3) = T_2 (S_2 - S_3) = Q_1$$

$$\text{Heat absorbed ( " } 4 \rightarrow 1) = T_1 (S_1 - S_4) = Q_2$$

$$\text{Since } S_3 = S_4 \text{ \& } S_2 = S_1 \Rightarrow Q_2 = T_1 (S_2 - S_3)$$

## \* Coefficient Of Performance (COP)

The performance ratio of refrigeration system is not the efficiency, but rather the Coefficient Of Performance, and define as the refrigeration effect (heat absorbed) divided by the net work done on the cycle (work input).

$$\text{COP}_R = \frac{Q_2}{W} = \frac{T_1 (S_2 - S_3)}{T_2 (S_2 - S_3) - T_1 (S_2 - S_3)} = \frac{T_1}{T_2 - T_1}$$

$$\text{COP}_R = \frac{T_{\text{Low}}}{T_{\text{High}} - T_{\text{Low}}}$$

It is more suitable to change the names of the processes of the reversed Carnot Cycle to:

|               |           |                                |                        |
|---------------|-----------|--------------------------------|------------------------|
| Heat absorbed | <u>to</u> | Refrigeration effect           | $Q_2 = T_1(S_2 - S_1)$ |
| Heat rejected | <u>to</u> | Heat rejected in the Condenser | $Q_1 = T_2(S_2 - S_1)$ |
| Work input    | <u>to</u> | Work input to Compressor       | $W = Q_1 - Q_2$        |

## \* Units of Refrigeration:

The practical unit of refrigeration is expressed in terms of "tonne of refrigeration" briefly written as "TR". A tonne of refrigeration is defined as the amount of refrigeration effect produced by the uniform melting of one tonne (1000 kg) of ice from and at 0°C in 24 hours. Since the latent heat of ice is 335 kJ/kg, therefore one tonne of refrigeration,

$$1 \text{ TR} = 1000 \times 335 \text{ kJ in 24 hours}$$

$$= \frac{1000 \times 335}{60 \times 24} = 232.6 \text{ kJ/min}$$

In actual practice, one tonne of refrigeration is taken as equivalent to 210 kJ/min or 3.5 kW (3.5 kJ/s).

**Ex:** A refrigerator has working temperature in the evaporator and condenser of -30°C and 32°C respectively, what is the maximum COP possible?, If the actual COP of 0.75 of the maximum COP, Calculate the refrigeration effect in kW per kW of power input.

**Sol:** 
$$\text{COP} = \frac{T_1}{T_2 - T_1} = \frac{-30 + 273}{(32 + 273) - (-30 + 273)} = 3.91$$

actual 
$$\text{COP}_r = 0.75 \times 3.91 = 2.939$$

$$\text{COP}_r = \frac{Q_2}{W} \Rightarrow 2.939 = \frac{Q_2}{1}$$

$$Q_2 = 2.939 \text{ kW of refrigeration / kW of work input.}$$

**Ex:** A machine working on a Carnot cycle operates between 305K and 260K. Determine the COP, when it is operated as:

1. a refrigerating machine,
2. a heat pump,
3. a heat engine.

**Sol:** 1. for refrigerating machine:

$$COP_R = \frac{T_1}{T_2 - T_1} = \frac{260}{305 - 260} = 5.78$$

2. for Heat pump:

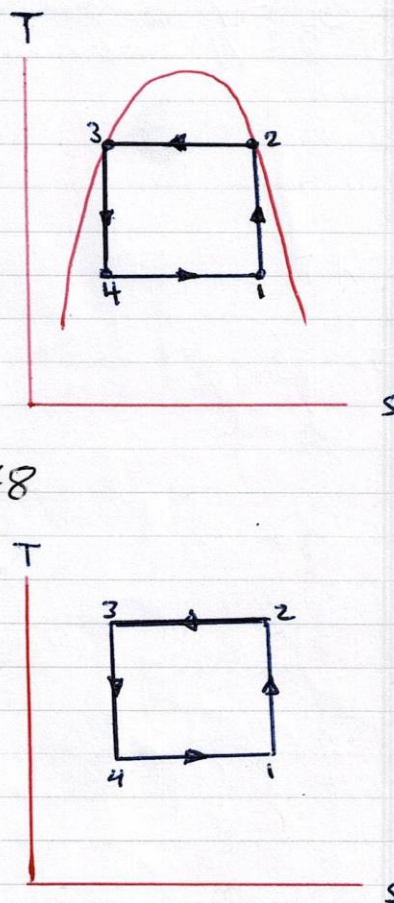
$$COP_P = \frac{T_2}{T_2 - T_1} = \frac{305}{305 - 260} = 6.78$$

$$\text{or } COP_P = COP_R + 1 = 5.78 + 1 = 6.78$$

3. for Heat Engine:

$$COP_E = \frac{T_2 - T_1}{T_2} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_{min}}{T_{max}}$$

$$\therefore COP_E = 1 - \frac{260}{305} = 0.147$$

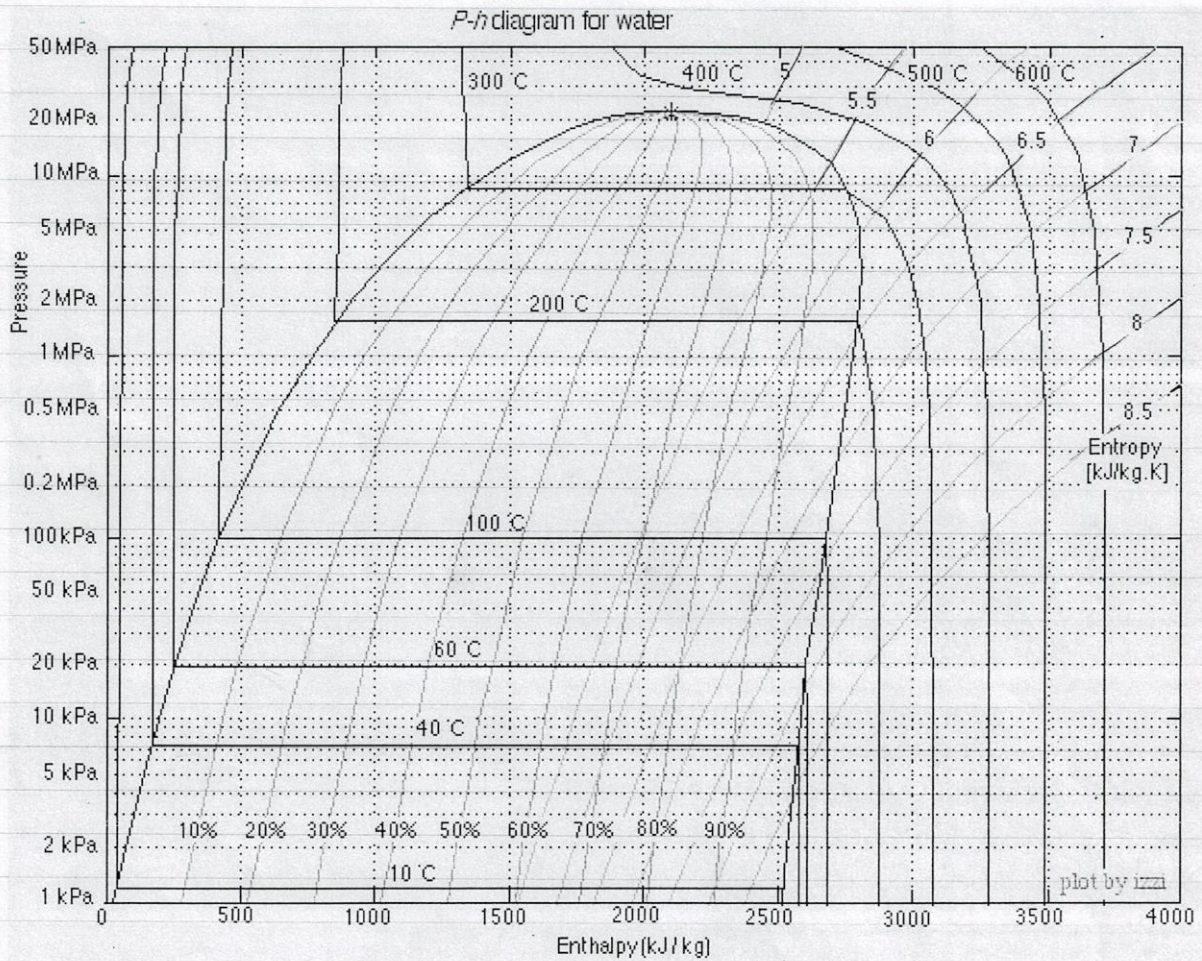


**H.W:** A Carnot refrigeration cycle absorbs heat at  $-3^\circ\text{C}$  and rejects it at  $27^\circ\text{C}$ .

1. Calculate the  $COP_R$ .
2. If the cycle is absorbing 1130 kJ/min at  $-3^\circ\text{C}$ , how many kJ of work is required per second?
3. If the Carnot heat pump operates between the same temperatures as the above refrigeration cycle, what is the  $COP_P$ ?
4. How many kJ/min will the heat pump deliver at  $27^\circ\text{C}$  if it absorbs 1130 kJ/min at  $-3^\circ\text{C}$ ?

## - Pressure - Enthalpy (p-h) Chart:

The most convenient chart for studying the behavior of a refrigerant is the p-h chart in which the vertical ordinates represent pressure and horizontal ordinates represent enthalpy.



## \* Types of Vapour Compression Cycle:

We have already discussed that a vapour compression cycle essentially consists of compression, condensation, throttling and evaporation. Many scientists have focussed their attention to increase the coefficient of performance of the cycle. Though there are many cycles, yet the following are important from the Subject point of view:

1. Cycle with dry saturated vapour after compression,
2. Cycle with wet vapour after compression,
3. Cycle with superheated vapour after compression,
4. Cycle with superheated vapour before compression, and
5. Cycle with undercooling or subcooling of refrigerant.

### \* 1 \* Vapour Compression Cycle with dry saturated Vapour after compression:

+ Process: 1 → 2: Compression process

$$W_{1-2} = h_2 - h_1 \quad \text{kJ/kg}$$

2 → 3: Condensing process

$$\text{heat rejected} = Q_1 = h_2 - h_3 \quad , \quad T_2 = T_3 \quad , \quad P_2 = P_3$$

$$= T_2 (S_2 - S_3) = T_2 \cdot S_{fg}$$

3 → 4: Expansion process  
(throttling process)

$$h_3 = h_4 \quad , \quad h_3 = h_f \text{ at } P_{\text{cond.}}$$

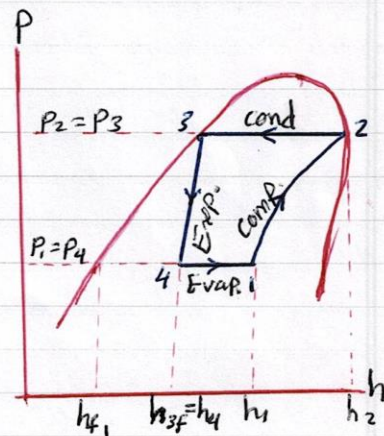
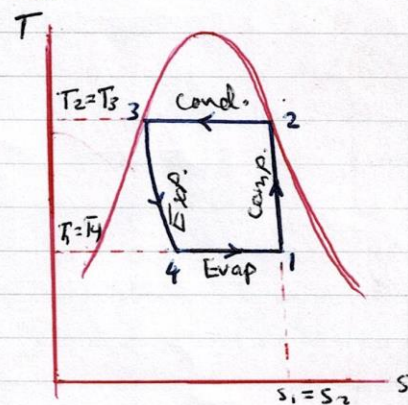
for Expansion process using Expansion valve or a capillary (small bore tube).

4 → 1: Vaporising process

$$RE = h_1 - h_4 \quad , \quad h_4 = h_3$$

$$= T_1 (S_1 - S_4)$$

$$\text{COP} = \frac{\text{Refrigerating Effect}}{\text{Work done}} = \frac{h_1 - h_4}{h_2 - h_1}$$





**Ex:** The temperature limits of an ammonia refrigerating system are  $25^{\circ}\text{C}$  and  $-10^{\circ}\text{C}$ . If the gas is dry saturated at the end of compression, calculate the coefficient of performance of the cycle assuming no undercooling of the liquid ammonia. Use the following table for properties of ammonia:

| Temperature ( $^{\circ}\text{C}$ ) | Liquid heat $\text{kJ/kg}$ | Latent heat $\text{kJ/kg}$ | Liquid entropy $\text{kJ/kg}\cdot\text{K}$ |
|------------------------------------|----------------------------|----------------------------|--------------------------------------------|
| 25                                 | 298.9                      | 1166.94                    | 1.1242                                     |
| -10                                | 135.37                     | 1297.68                    | 0.5443                                     |

**Sol:**  $S_1 = S_f + x_1 S_{fg}$  ,  $h_{fg_{4-1}} = T_1 \cdot S_{fg} \Rightarrow S_{fg} = \frac{h_{fg}}{T_1}$

$$S_1 = 0.5443 + x_1 \cdot \frac{1297.68}{263} = 0.5443 + 4.934 x_1$$

$$S_2 = S_g \text{ at } T_{\text{cond.}} , S_2 - S_3 = S_g - S_f$$

$$h_2 - h_3 = h_{fg_{\text{cond.}}} = T_2 \cdot S_{fg_{\text{cond.}}} \Rightarrow S_2 = S_f + \frac{h_{fg_{\text{cond.}}}}{T_2}$$

$$= 1.1242 + \frac{1166.94}{298}$$

$$= 5.04 \text{ kJ/kg}\cdot\text{K} = S_1$$

$$\therefore 5.04 = 0.5443 + 4.934 \cdot x_1 \Rightarrow x_1 = 0.91$$

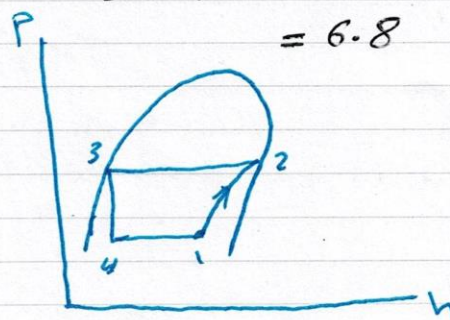
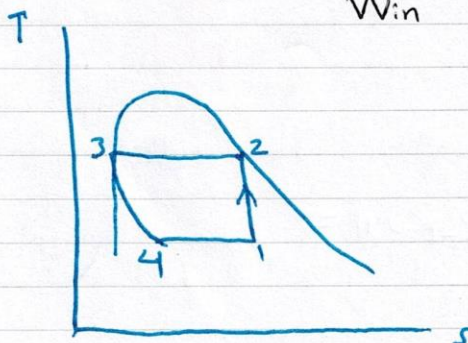
$$h_1 = h_{f_1} + x_1 h_{fg_1} = 135.37 + 0.91 \cdot 1297.68 = 1316.26 \text{ kJ}$$

$$h_2 = h_g = h_{fg_2} + h_{f_2} = 298.9 + 1166.94 \text{ kJ/kg}$$

$$h_4 = h_3 = h_{f_{\text{cond.}}} = h_{f_3}$$

$$\text{COP}_R = \frac{R_E}{W_{\text{in}}} = \frac{h_1 - h_{f_3}}{h_2 - h_1} = \frac{1316.26 - 298.9}{1465.84 - 1316.26}$$

$$= 6.8$$

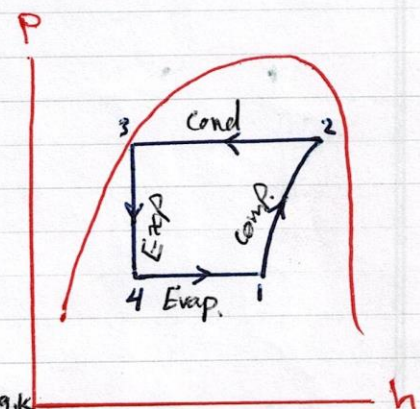
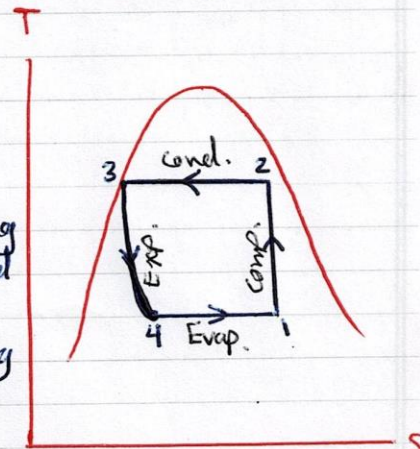


## 2. Vapours Compression Cycle with Wet Vapour after Compression :

$$\text{COP} = \frac{R.E.}{W_{in}} = \frac{h_1 - h_{f3}}{h_2 - h_1}$$

Ex: Fin the COP for a CO<sub>2</sub> machine working between the temperature range of 25°C and -5°C. The dryness fraction of CO<sub>2</sub> gas during the suction stroke is 0.6. Following properties of CO<sub>2</sub> are given :

| T<br>°C | h <sub>f</sub><br>kJ/kg | h <sub>fg</sub><br>kJ/kg | h <sub>g</sub><br>kJ/kg | S <sub>f</sub><br>kJ/kg.K | S <sub>g</sub><br>kJ/kg.K |
|---------|-------------------------|--------------------------|-------------------------|---------------------------|---------------------------|
| 25      | 81.3                    | 121.4                    | 202.6                   | 0.251                     | 0.63                      |
| -5      | -7.54                   | 245.3                    | 237                     | -0.042                    | 0.84                      |



Sol

$$S_1 = S_{f1} + x_1 S_{fg} \Rightarrow S_{fg1} = \frac{h_{fg}}{T_1}$$

$$= -0.042 + \frac{0.6 \times 245.3}{268} = 0.507 \text{ kJ/kg.K}$$

$$S_2 = S_{f2} + x_2 \cdot \frac{h_{fg2}}{T_2} = S_1$$

$$\therefore 0.507 = 0.251 + x_2 \cdot \frac{121.4}{298} \Rightarrow x_2 = 0.629$$

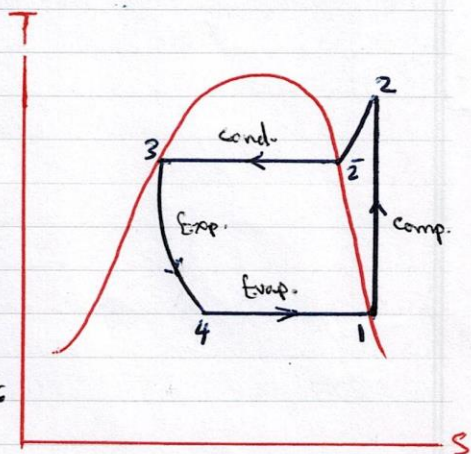
$$h_1 = h_{f1} + x_1 h_{fg1} = -7.54 + 0.6 \times 245.3 = 139.64 \text{ kJ/kg}$$

$$h_2 = h_{f2} + x_2 h_{fg2} = 81.3 + 0.629 \times 121.4 = 157.66 \text{ kJ/kg}$$

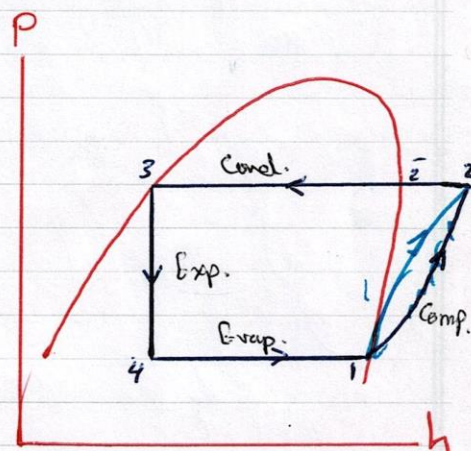
$$\therefore \text{COP} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{139.64 - 81.3}{157.66 - 139.64} = 3.24$$

### 3. Vapour Compression Cycle with Superheated Vapour after Compression

Ex: A vapour compression refrigerator uses methyl chloride (R40) and operating between temperature limits of  $-10^{\circ}\text{C}$  and  $45^{\circ}\text{C}$ . At entry to the compressor, the refrigerant is dry saturated and after compression it acquires a temperature of  $60^{\circ}\text{C}$ . Find the COP of refrigerator.



| $T_{\text{sat.}} (^{\circ}\text{C})$ | $h_f$<br>kJ/kg | $h_g$<br>kJ/kg | $S_f$<br>kJ/kg.K | $S_g$<br>kJ/kg.K |
|--------------------------------------|----------------|----------------|------------------|------------------|
| -10                                  | 45.4           | 460.7          | 0.183            | 1.637            |
| 45                                   | 133.0          | 483.6          | 0.485            | 1.587            |



Sol:

$$s_2 - s_2' = C_p \ln \frac{T_2}{T_2'}$$

$$s_2 = s_2' + C_p \ln \frac{T_2}{T_2'} = s_1$$

$$1.637 = 1.587 + C_p \ln \frac{333}{318}$$

$$\therefore C_{p_{\text{sup}}} = 1.09 \text{ kJ/kg.K}$$

$$h_2 = h_2' + C_p \times \text{D.O.S}$$

, D.O.S  $\equiv$  Degree of Superheat

$$= h_2' + C_p (T_2 - T_2')$$

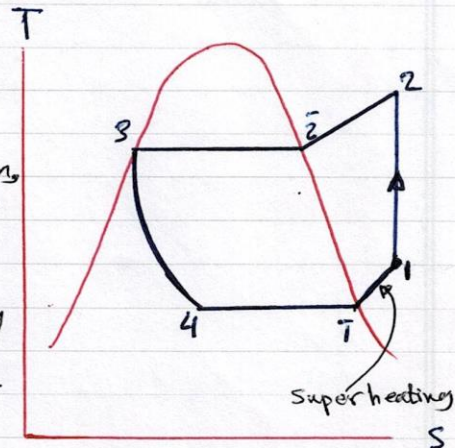
$$h_2 = 483.6 + 1.09 (333 - 318) = 500 \text{ kJ/kg}$$

$$\therefore \text{COP} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{460.7 - 133}{500 - 460.7} = 3.77$$

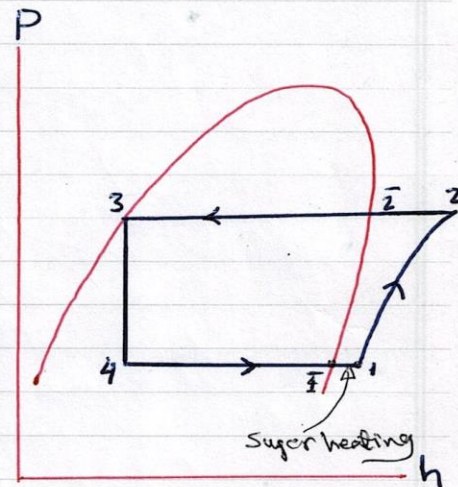
## 4. Vapour Compression Cycle with Superheated Vapour before Compression:

**Ex:** A vapour compression refrigeration plant works between pressure limits of 5.3 bar and 2.1 bar. The vapour is superheated at the end of compression, its temperature being 37°C. The vapour is superheated by 5°C before entering the compressor.

If the specific heat of superheated vapour is 0.63 kJ/kg.K, find the COP of the plant. Use data given below:



| Pressure (bar) | $T_{sat}$ (°C) | $h_f$ (kJ/kg) | $h_{fg}$ (kJ/kg) |
|----------------|----------------|---------------|------------------|
| 5.3            | 15.5           | 56.15         | 144.9            |
| 2.1            | -14.0          | 25.12         | 158.7            |



Sol

$$h_1 = h_i + c_{p_v}(T_1 - T_i) \quad \text{and} \quad h_i = h_g$$

$$h_i = h_{fg} + h_f$$

$$\therefore h_1 = (25.12 + 158.7) + 0.63 \times (5)$$

$$= 186.97 \text{ kJ/kg}$$

$$h_2 = h_{\bar{2}} + c_{p_v}(T_2 - T_{\bar{2}}) = (h_{f_{\bar{2}}} + h_{fg_{\bar{2}}}) + c_{p_v}(T_2 - T_{\bar{2}})$$

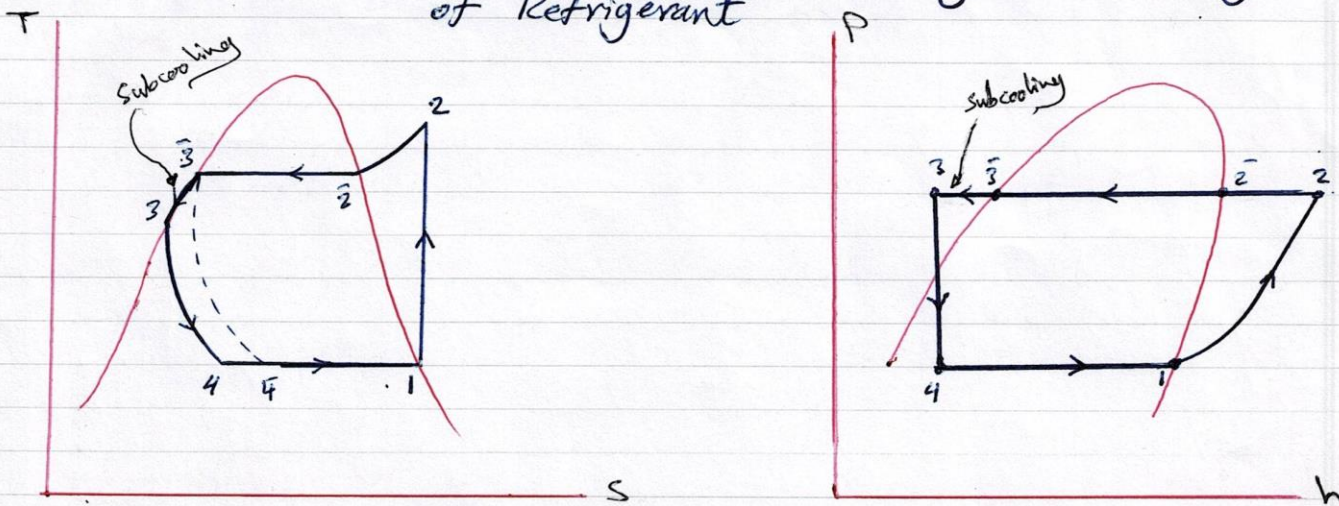
$$\therefore h_2 = (56.15 + 144.9) + 0.63 \times (37 - (-14))$$

$$= 214.6 \text{ kJ/kg}$$

$$COP = \frac{R.E}{W_{in}} = \frac{h_1 - h_{f3}}{h_2 - h_1} = \frac{186.97 - 56.15}{214.6 - 186.97}$$

$$\therefore COP = 4.735$$

## 5. Vapour Compression Cycle with undercooling or subcooling of Refrigerant



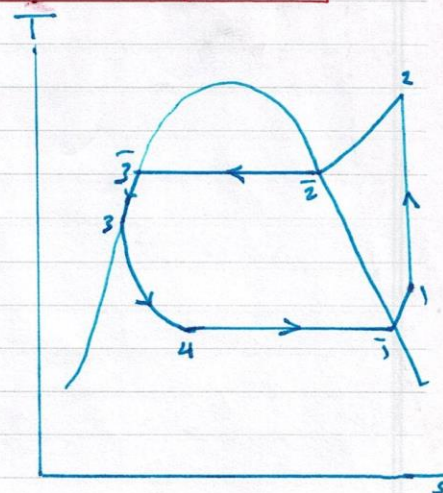
**Ex:** A food storage locker requires a refrigeration capacity of 12TR, and works between the evaporating temperature of  $-8^{\circ}\text{C}$  and condensing temperature of  $30^{\circ}\text{C}$ . The refrigerant R12 is subcooled by  $5^{\circ}\text{C}$  before entry to expansion valve and the vapour is superheated to  $-2^{\circ}\text{C}$  before leaving the evaporator coils. Determine: 1. COP 2. Power per TR.  
Use the following data for R-12:  $C_p = 1.235 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$ ,  $C_v = 0.733$

| Temp ( $^{\circ}\text{C}$ ) | Pressure (bar) | Enthalpy kJ/kg |        | Entropy kJ/kg.K |        |
|-----------------------------|----------------|----------------|--------|-----------------|--------|
|                             |                | $h_f$          | $h_g$  | $S_f$           | $S_g$  |
| -8                          | 2.354          | 28.72          | 184.07 | 0.1149          | 0.7007 |
| 30                          | 7.451          | 64.59          | 199.62 | 0.2400          | 0.6853 |

Sol  $\underline{1.}$   $S_1 = S_f + C_p \ln \frac{T_1}{T_f}$   
 $= 0.7007 + 0.733 \ln \frac{271}{265} \Rightarrow$

$S_1 = 0.7171 \text{ kJ/kg}\cdot\text{K}$

$S_2 = S_g + C_p \ln \frac{T_2}{T_g}$   
 $= 0.6853 + 0.733 \times \ln \frac{T_2}{303} = S_1$



$$\circ \circ \quad 0.7171 = 0.6853 + 0.733 \ln \frac{T_2}{303}$$

$$\circ \circ \quad T_2 = 316.4 \text{ K} \quad \text{or} \quad 43.4^\circ \text{C}$$

$$h_1 = h_i + C_{p_v} (T_1 - T_i)$$

$$= 184.07 + 0.733 (271 - 265) = 188.47 \text{ kJ/kg}$$

$$h_2 = h_{\bar{2}} + C_{p_v} (T_2 - T_{\bar{2}})$$

$$= 199.62 + 0.733 (316.4 - 303) = 209.44 \text{ kJ/kg}$$

$$Q_{3 \rightarrow \bar{3}} = C_p (T_{\bar{3}} - T_3) = h_{\bar{3}} - h_3 \quad \rightarrow \quad h_{\bar{3}} = h_f$$

$$h_3 = h_{f_{\bar{3}}} - C_{p_l} (T_{\bar{3}} - T_3)$$

$$= 64.59 - 1.235 * 5$$

$$= 58.42 \text{ kJ/kg} = h_4$$

$$\text{COP} = \frac{\text{R.E}}{W_{\text{in}}} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{188.47 - 58.42}{209.44 - 188.47} = 6.2$$

2.  $\text{R.E} = h_1 - h_4 = 188.47 - 58.42 = 130.05 \text{ kJ/kg}$

Refrigerating Capacity ( $Q$ ) = 12 TR

$$= 12 * 210 = 2520 \text{ kJ/min}$$

or  $Q = 12 * 3.5 = 42 \text{ kW}$

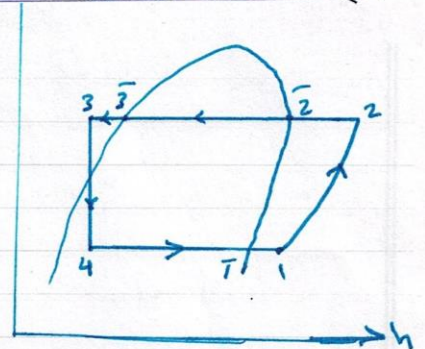
$$Q = \dot{m} * \text{R.E} \Rightarrow \dot{m} = \frac{Q}{\text{R.E}} = \frac{42}{130.05} = 0.32295 \text{ kg/s}$$

$$\text{Power} = W_{\text{in}} = \dot{m} (h_2 - h_1) = 0.32295 * (209.44 - 188.47) = 6.7723 \text{ kW}$$

$$\text{Power per tonne of Refrigeration} = \frac{W}{Q}$$

$$= \frac{6.7723}{12}$$

$$= 0.564 \frac{\text{kW}}{\text{TR}}$$



## „ Sheet No. 9 „

Q1: The temperature in evaporator coils is  $-6^{\circ}\text{C}$  and that in the condenser coil is  $22^{\circ}\text{C}$ . Assuming that the machine operates on the reversed Carnot cycle. Calculate the COP, the Refrigeration effect per kW of input work, and the heat rejected to the condenser.

Ans. (9.54, 9.54, 10.4 kW)

Q2: A vapour compression refrigerator using R134a works between temperature limits of  $-5^{\circ}\text{C}$  and  $40^{\circ}\text{C}$ . The refrigerant leaves the compressor dry saturated. Calculate the refrigeration effect and COP if, a. the refrigerant leaves the condenser saturated, and b. the refrigerant is subcooled to  $20^{\circ}\text{C}$  before entering the throttle valve.

Ans. (a) 131.15 kJ/kg, 4.54, b) 163.08 kJ/kg, 5.65

Q3: In a refrigerator, R134a is compressed isentropically from a saturated state at  $-5^{\circ}\text{C}$  to a pressure of 11.59 bar. The refrigerant is then cooled at constant pressure to  $25^{\circ}\text{C}$ , and is throttled down to a temperature of  $-5^{\circ}\text{C}$  at which it is evaporated. Determine the temperature and enthalpy after compression in two ways: (a) by using the tables (b) by assuming the  $C_{p,v} = 1.153 \text{ kJ/kg}\cdot\text{K}$ . Sketch the cycle on T-S and p-h diagrams and calculate the COP by each method.

Ans. (a)  $50.66^{\circ}\text{C}$ , 428.05 kJ/kg, 4.94, b)  $50.62^{\circ}\text{C}$ , 428.01, 4.95

Q4: An ammonia refrigerating machine fitted with an expansion valve works between the temperature limits of  $-10^{\circ}\text{C}$  and  $30^{\circ}\text{C}$ . The vapour is 95% dry at the end of isentropic compression and the fluid leaving the condenser is at  $30^{\circ}\text{C}$ . If the actual coefficient of performance is 60% of the maximum, find the ice produced per kW hour at  $0^{\circ}\text{C}$  from water at  $10^{\circ}\text{C}$ . The latent heat of ice is 335 kJ/kg. The ammonia has the following properties:

| Temperature ( $^{\circ}\text{C}$ ) | $h_f$ (kJ/kg) | $h_{fg}$ (kJ/kg) | $S_f$  | $S_g$  |
|------------------------------------|---------------|------------------|--------|--------|
| 30                                 | 323.08        | 1145.79          | 1.2037 | 4.9842 |
| -10                                | 135.37        | 1297.68          | 0.5443 | 5.477  |

Ans. (33.1 kg/kW.h)